

Experiment 1 – Fraunhofer and Fresnel Diffraction: Diffraction Patterns of a Circular Aperture and a Straight Edge

References:

These references contain good discussions of scalar diffraction theory:

Optics by Eugene Hecht, Chapter 10

Introduction to Modern Optics by Grant Fowles, Chapter 5

Principles of Optics by Max Born and Emil Wolf, Chapter 8

The following reference contains a superb treatment of Gaussian spherical beams:

An Introduction to Lasers and Masers by A.E. Siegman, Chapter 8

Introduction:

This experiment will extend your knowledge of Fraunhofer diffraction and introduce you to situations in which the more general Fresnel diffraction calculations are required. In Section I you will measure the Fraunhofer diffraction pattern produced by a circular aperture and compare your data with the predictions of scalar diffraction theory. In Section II you will spatially filter a helium-neon laser beam and study the famous Fresnel diffraction pattern of a straight edge.

In both parts of the experiment, you will want to begin by simply looking at the diffraction pattern you are attempting to measure. Then become familiar with the measurement capabilities and limitations of the experimental setup. With this knowledge you can then think and plan carefully to obtain the best possible measurements of your diffraction patterns in a reasonable amount of time.

Once you have taken your data appropriately (in Section I or in Section II), this experiment affords an excellent opportunity to witness the power of error analysis and non-linear chi-square fitting. Take your results seriously and expect to obtain good fits with reasonable chi-square values and random residuals. If you don't achieve them, you should investigate and understand any effects which keep you from getting them!

I. Fraunhofer Diffraction Pattern of a Circular Aperture

You should read carefully an account of the calculation of the Fraunhofer diffraction pattern produced by a plane wave incident upon a circular aperture. Most introductory optics texts include this calculation, and you should write your own version in your notebook. The references listed above have good treatments of scalar diffraction theory, although you are encouraged to consult any of the other optics texts available to you.

We would like to test the prediction that the intensity in the diffraction pattern is given by

$$I(\theta) = 4I_{\max} \frac{[J_1(ka \sin \theta)]^2}{(ka \sin \theta)^2} \quad (1-1)$$

where θ is the angle of diffraction, $k = 2\pi/\lambda$, a is the *radius* of the aperture, and J_1 is the Bessel function of first order. The experimental setup includes a 5 mW helium-neon laser (JDS Uniphase Model 1125P, $\lambda = 632.8$ nm), a circular aperture with a *diameter* of roughly 200 μm , and a photodiode detector with amplifier. **Do not touch the circular aperture; finger grease and dead skin cells severely degrade the quality of the diffraction pattern.** With this equipment you can record the intensity at a number of points in the diffraction pattern, and then fit the data to the functional form of Eqn. (1-1). Appendix A at the end of the lab manual describes how to perform the non-linear least squares fit to the circular aperture data using the software package Origin in the PC Windows environment. Similarly, instructions for using IGOR Pro can be found in Appendix D.

Start by directing the laser beam onto the 200 μm diameter aperture. (See Fig. 1-1.) The diffraction aperture is mounted on an optical rail carrier with horizontal and vertical translation, so the aperture can be centered in the laser beam. At the output of the laser, the beam forms a waist (Gaussian spherical beam waist) with a spot diameter of $2w_0 = 0.83$ mm. In other words, the surface of constant phase of the electric field is a plane, and a circular aperture with a diameter of 0.83 mm will pass 86% of the beam power. The intensity of the beam falls off in the transverse direction (*i.e.*, perpendicular to the beam propagation) as $\exp(-2r^2/w_0^2)$ where r is the distance from the center axis of the beam. The finite diameter of the beam effectively forms an aperture through which the beam is continuously diffracting, so the beam is often described as “diffraction-limited.” This process of diffraction leads to divergence of the beam as it propagates away from the laser output (in our case, along the optical rail); the beam divergence half-angle is $\theta_{1/2} = \lambda/\pi w_0 = 0.50$ mrad. Most treatments of Fraunhofer diffraction from a circular aperture assume a constant intensity over the aperture. You will have to decide what distance s you want between the laser and the diffraction aperture so that the intensity over the area of the aperture is roughly constant.

There are other considerations in arranging the experimental setup. The distance s from laser to diffraction aperture and the distance r from diffraction aperture to the photodiode detector should be chosen so that the following *Fraunhofer condition* is satisfied:

$$s/d \gg d/\lambda \quad \text{and} \quad r/d \gg d/\lambda \quad (1-2)$$

where $d = 2a$ is the diameter of the diffraction aperture. When this condition is satisfied (a ratio of 10 should do) the diffraction pattern formed on the detector aperture is the Fraunhofer pattern. The detector aperture (the aperture immediately in front of the photodiode) must be chosen judiciously. Larger apertures allow more light to hit the photodiode and hence give more signal, but they average over a larger area of the diffraction pattern. (Note that since the active area of the photodiode is roughly 2.5 mm in diameter, there is no point in using a detector aperture with a diameter larger than

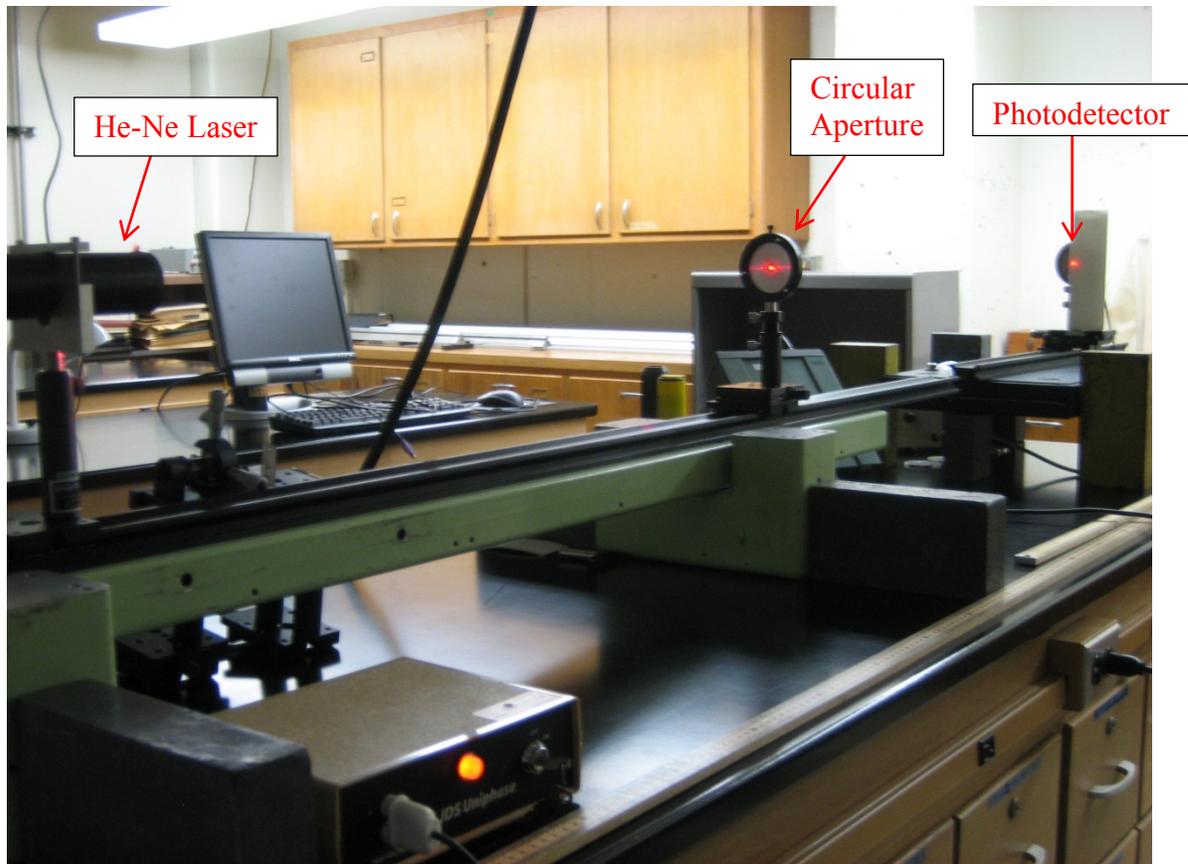


Fig. 1-1 The experimental setup for measuring the Fraunhofer diffraction pattern of a circular aperture. The He-Ne laser is on the left, and its red beam impinges upon the 200- μm diameter circular aperture just to the right of the center of the photo. The diffraction pattern of the aperture is partially displayed on a white sheet of paper at the location of the photodetector.

2.5 mm.) You should choose a detector aperture which is the optimal compromise between too little signal and too much spatial averaging. Of course, increasing the distance r between the diffraction aperture and photodiode also reduces the effect of spatial averaging and decreases the overall intensity on the photodiode.

Record the intensity at an appropriate number of points in the diffraction pattern. Uncertainties in the data points should be determined by forming a sample variance of several scans through the pattern.

The dependence of $I(\theta)$ in Eqn. (1-1) on the fitting parameter a is certainly not linear. Hence, though you have written (or will write) your own linear least squares fitting routine as part of this course, you will not be able to use your routine for this experiment. However, Bevington and Robinson (our required text, Section 8.6) describe a computer program MARQUARDT which performs least squares fits to fitting functions which are nonlinear functions of fitting parameters. MARQUARDT is based on an algorithm published in 1963 by D.W. Marquardt (*J. Soc. Ind. Appl. Math.* **2**: 431-441 (1963)). This same algorithm, often called the Levenberg-Marquardt algorithm, is the basis of the

nonlinear least squares fitting routines in the commercially available software packages Igor Pro (for MACs and PCs) and Origin (for PCs). It is also available as a fitting algorithm for the MATLAB non-linear least squares fitting routine LSQCURVEFIT. The Appendices at the end of the lab manual contain tutorials for these software packages. Origin's and Igor's built-in math libraries contain Bessel functions which can be used in the fitting function, and MATLAB also has Bessel functions in its math library.

The instructions in the Appendices will lead you step-by-step through the mechanics of fitting your data. The main challenge for you will be to apply your understanding of χ^2 fitting and error analysis in order to guide the fitting routine most efficiently and interpret its output sensibly.

Don't be upset if your value for the radius a of the aperture deviates from 100 μm . This value is a nominal value specified by the manufacturer with a tolerance of 10%. Concentrate on obtaining a good, self-consistent fit.

II. Fresnel Diffraction by a Straight Edge

Read carefully an account of the calculation of the Fresnel diffraction pattern of a straight edge. Most optics texts treat this classic example; many students find the treatment in the reference by Fowles particularly clear. Astrophysicists measure the Fresnel diffraction pattern produced on the earth's surface by starlight as it passes the edge of the moon's disk (called lunar occultation). Deviations from the expected pattern for a point source enable them to deduce a value for the diameter of the star. The disk of the star is otherwise unresolvable by earth-bound telescopes.

You can create a point source with a substantial cone of light by spatially filtering the laser beam. The spatial filter consists of a microscope objective and a circular aperture, housed together. The microscope objective produces a focused waist of the laser beam shortly beyond the objective. The aperture transmits only the waist of the Gaussian spherical laser beam, and blocks stray reflections and imperfections in the beam. The final result is a beautiful cone of light with no peripheral imperfections – hence the name “spatial filter”. Chapter 8 of the reference by Siegman contains an excellent description of Gaussian spherical beams.

Mount the spatial filter on the optical rail carrier with horizontal and vertical translation. (See Fig. 1-2.) Remove the 25 μm diameter aperture from the spatial filter and direct the laser beam through the microscope objective, using the horizontal and vertical translation of the carrier to center the objective on the laser beam. Replace the 25 μm diameter aperture and use the horizontal and vertical micrometers on the spatial filter to maximize the intensity transmitted by the aperture. You may need to use the axial screw adjustment on the spatial filter to translate the microscope objective slightly forwards or backwards along the beam path so that its focal plane coincides with the

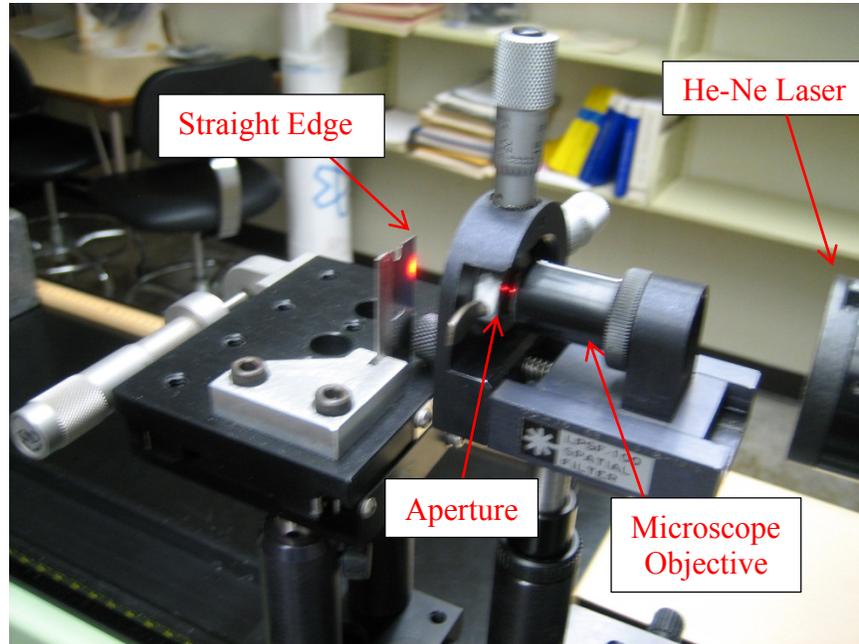


Fig. 1-2 A spatial filter consists of a focusing lens (microscope objective) and a circular aperture. The laser beam is focused through the aperture while stray light from the laser resonator is caught by the edges of the aperture, leaving a very smooth and uniform beam incident upon the straight edge.

plane of the $25\ \mu\text{m}$ aperture. When you have positioned the focused “waist” of the laser beam in the center of the $25\ \mu\text{m}$ aperture, the transmitted intensity will be a maximum.

Direct the spatially filtered beam onto the razor blade which is fastened to a micrometer-driven translator which in turn is mounted on an optical rail carrier. Rather than translate the photodiode to scan the diffraction pattern, you can simply translate the razor blade using the micrometer. This procedure keeps the photodiode in the same portion of the spatially filtered beam. Use your ingenuity to devise a convenient geometry of spatial filter, razor blade, and detector. Compare your data with the predictions of Fresnel diffraction theory. Fabio Rojas and David Painter (both '96) and Kevin Moore ('99) have written a graphical user interface (GUI) for MATLAB that facilitates the fitting of straight edge data. See Appendix C at the end of this lab manual.

Some students in the past have become intrigued with the famous Poisson bright spot in the diffraction pattern of a ball bearing. In fact, if you point the spatially filtered beam at just about anything in the lab you'll see some fairly remarkable wave phenomena. You are certainly free to test any of the predictions of diffraction theory in addition to or instead of the examples suggested above, but be sure to clear your plans with your instructor.

III. Recording 2-D Diffraction Patterns with a CCD Camera

During the summer of 2012, the Physics Department purchased a CCD camera from Thorlabs (DCU224M, 1280×1024 pixels, B&W, sensitive area $5.95 \text{ mm} \times 4.76 \text{ mm}$, pixel area $4.65 \text{ }\mu\text{m} \times 4.65 \text{ }\mu\text{m}$, frame rate 15 fps, purchase price \$2400). The camera communicates with a host computer via a USB 2.0 cable. This camera is available to us for recording 2-D diffraction patterns at speeds much, much faster than is possible by recording them point-by-point manually. And, of course, we can have a lot of fun fitting 2-D diffraction patterns or 1-D slices through these 2-D patterns with functions derived from Fraunhofer and Fresnel diffraction theory.

In Optics Lab during the spring of 2013, Peter Megson and Jaron Kent-Dobias (both HMC '2014) used a red LED and crossed polarizers to demonstrate that the 1.3 megapixels of the DCU224M CCD camera respond linearly to incident light, and that the pixels are remarkably uniform across the entire array. (The tech reports of Peter and Jaron are available on the Sakai course site.) There is just one catch. A protective glass coverslip over the array results in an annoying interference pattern when a light source like a He-Ne laser (with coherence length much longer than the thickness of the coverslip) is used to illuminate the camera. Jaron was able to reduce the effect of the interference pattern by a judicious normalization procedure, but we may want to explore ways to avoid the interference pattern altogether. For example, we could display a diffraction pattern of interest on a white screen and record the back-scattered light from the screen with the CCD camera. Or perhaps we could display the diffraction pattern of interest on a ground-glass screen and position the CCD camera to image the forward-scattered light transmitted through the ground-glass screen to the camera.

There clearly is enormous potential for recording and fitting unusual 2-D diffraction patterns if we can successfully employ the CCD camera to record those patterns without artifacts. If you are interested in participating in the successful implementation of the camera to record and fit 2-D diffraction patterns, just chat with your instructor about how you can contribute to the mission! Work on the camera may replace some parts of Sections I and II if that seems appropriate, and there are certainly many opportunities here for ground-breaking technical report work!