Experiment 5 - Fourier Transform Spectroscopy

References: Optics by Miles Klein and Thomas Furtak, Chapter 6, Sections 1 and 2
Introductory Fourier Transform Spectroscopy by Robert J. Bell
Optics by Eugene Hecht. Chapter 12 contains an introduction to coherence theory.
**Fundamentals of Optics by F. Jenkins & H. White, Chapter 13, Sections 8 through 15, and Chapter 14, Section 13 – *required reading!*

Traditional dispersive techniques for observing the spectrum of a light beam have employed gratings (dispersion by diffraction) and prisms (dispersion by refraction). In the more recently developed technique of Fourier transform spectroscopy, the output intensity $I(\tau)$ of a Michelson interferometer is measured as a function of the optical path difference $2d$ between the two legs of the instrument ($\tau = 2d/c$). The Fourier transform of this output intensity yields the spectrum of the light beam incident on the interferometer. The relationship can be expressed as

$$P(\omega) = \frac{2}{\pi}\int_0^\infty \left[\frac{I(\tau)}{< I >} - 1\right]\cos\omega\tau\, d\tau \qquad (5\text{-}1)$$

where $P(\omega)$ is the normalized spectral distribution of the light beam incident on the interferometer, i.e.,

$$\int_0^\infty P(\omega) d\omega = 1$$

$I(\tau)$ is the output intensity of the Michelson interferometer when the optical path difference between the two legs of the instrument is $2d$, or when the difference in time of light propagation is $\tau = 2d/c$. $<I>$ is the average output intensity over the full range of τ. The integral in Eq. (5-1) is the Fourier cosine transform. The inverse Fourier cosine transform relationship is

$$\left[\frac{I(\tau)}{< I >} - 1\right] = \int_0^\infty P(\omega)\cos\omega\tau\, d\omega \qquad (5\text{-}2)$$

Eqs. (5-1) and (5-2) express the fact that $P(\omega)$ and $[I(\tau)/<I> - 1]$ comprise a Fourier cosine transform pair.

A Michelson interferometer that has been modified and fine-tuned for this technique is called a Fourier transform spectrometer (see Figure 5-2), and the resolving power of the spectrometer depends in principle on the maximum optical path length difference that can be introduced between the two legs of the instrument. Fourier transform spectrometers with half-meter mirror movements have been built with resulting resolving powers on the order of 10^6, which is truly state-of-the-art. With such an instrument, and sufficiently fine sampling of the intensity vs. path length difference, a reliable spectrum can be obtained by full Fourier transform techniques.

Often the general shape of the spectrum of the incident beam is known, but the magnitude of a particular spectral parameter is unknown. For example, the incident spectrum may consist of two spectral lines of roughly equal intensity whose spacing is to be measured, e.g., the yellow doublet of the sodium discharge lamp. Or the incident spectrum may consist of a single line with Lorentzian shape whose half-width-at-half-maximum (HWHM) is to be measured, e.g., the pressure broadened green line of the mercury discharge lamp. In these cases the explicit Fourier transform of the output intensity data need not be calculated, since the functional forms of the relevant Fourier transform pairs are well known. For the sodium doublet, $P(\omega)$ may be written approximately as the sum of two equally weighted delta functions

$$P(\omega) = \tfrac{1}{2}\delta\left(\omega - \omega_{ave} - \frac{\Delta\omega}{2}\right) + \tfrac{1}{2}\delta\left(\omega - \omega_{ave} + \frac{\Delta\omega}{2}\right) \tag{5-3}$$

where $\Delta\omega$ is the doublet separation, and ω_{ave} is the average angular frequency of the doublet. The corresponding output intensity $I(\tau)$ is then

$$I(\tau) = < I > \left[1 + \cos\left(\frac{\Delta\omega}{2}\tau\right)\cos\left(\omega_{ave}\tau\right)\right] \tag{5-4}$$

You should verify that Eqs. (5-3) and (5-4) are consistent with Eqs. (5-1) and (5-2).*

For the pressure-broadened green line of the mercury discharge lamp, $P(\omega)$ may be written as the Lorentzian spectral profile:

$$P(\omega) = \frac{1}{\pi}\frac{(1/\tau_p)}{(\omega - \omega_0)^2 + (1/\tau_p)^2} \tag{5-5}$$

where the HWHM is $(1/\tau_p)$, and ω_0 is the angular frequency corresponding to $\lambda = 546$ nm. The parameter τ_p is the mean time between collisions of mercury atoms. The corresponding $I(\tau)$ is

$$I(\tau) = < I > \left\{1 + \exp\left(-\tau/\tau_p\right)\cos\left(\omega_0\tau\right)\right\} \tag{5-6}$$

Again you should verify that Eqs. (5-5) and (5-6) are consistent with Eqs. (5-1) and (5-2). Hints: when evaluating the integral in Eq. (5-1), use the first identity in the footnote and then express (for example) $\cos(\omega - \omega_0)\tau$ as $\mathrm{Re}(\exp(i(\omega - \omega_0)\tau))$. The resulting integral is straight-forward.

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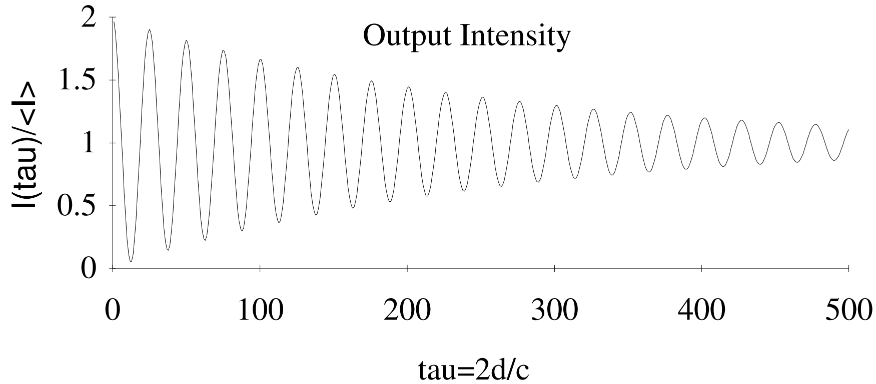
* You'll find the following identities useful:

$$2\cos\omega_1\tau\,\cos\omega_2\tau = \cos(\omega_1 + \omega_2)\tau + \cos(\omega_1 - \omega_2)\tau \qquad \delta(\omega - \omega_0) = \frac{2}{\pi}\int_0^\infty \cos\omega_0\tau\,\cos\omega\tau\,d\tau$$

Please note, however, that the integral in Eq. (5-1) yields two terms, one of which is always negligibly small and does not appear in Eq. (5-5). When evaluating the integral in Eq. (5-2), note that the limits can be extended to $-\infty$ to $+\infty$ because the width of the line ($1/\tau_p$) is many orders of magnitude smaller than the center frequency ω_0. Now the integral becomes a great application of the residue theorem from complex analysis!



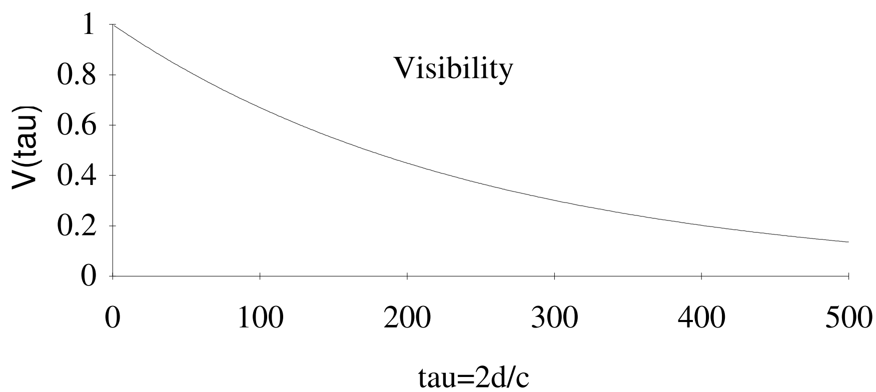


Figure 5-1. Representative plots of output intensity and visibility for a Lorentzian spectral line.

In both of the examples in Eqs. (5-4) and (5-6), the output intensity $I(\tau)$ consists of a rapidly fluctuating fringe term, $\cos \omega_{ave}\tau$ or $\cos \omega_0\tau$, which is modulated by a slow envelope, $\cos\left(\frac{\Delta\omega}{2}\tau\right)$ or $\exp\left(-\frac{\tau}{\tau_p}\right)$, respectively. In these cases and in other similar situations, the value of a spectral parameter can be obtained simply by measuring the so-called visibility $V(\tau)$ of the fringes as a function of mirror position. The visibility of the fringes is defined to be the zero-to-peak amplitude of the fast fringes divided by the average output intensity $<I>$, i.e., just the magnitude of the slow envelope of the fringes. Hence for the sodium doublet and the mercury green line, the visibilities $V(\tau)$ are $\left|\cos\left(\frac{\Delta\omega}{2}\tau\right)\right|$ and $\exp\left(-\frac{\tau}{\tau_p}\right)$, respectively.

Figure 5-1 illustrates $I(\tau)$ and $V(\tau)$ for the mercury green line. Measurements of $V(\tau)$ for these two cases will therefore yield directly the spacing of the sodium doublet and the mean time between collisions of mercury atoms.

Part I - Measurement of the sodium doublet separation.

The experimental apparatus consists of a sodium discharge lamp, an optical interference filter that eliminates all spectral lines except the yellow doublet, a homemade Fourier transform spectrometer, a photodiode and amplifier, an electrical filter, and a digital oscilloscope (see Figure 5-2). The electrical filter is necessary to suppress the 120 Hz (and higher harmonics) fluctuation of the lamp intensity due to the 60 Hz stimulating voltage. Center the circular fringes formed at the output of the spectrometer on the photodiode. If you don't see the fringes at the output, try translating the mirror to make sure the visibility of the fringes, $\left|\cos\left(\frac{\Delta\omega}{2}\tau\right)\right|$, doesn't simply happen to be zero. If you still don't see fringes, use the following steps to bring the instrument into proper adjustment:

1. Use a ruler to position the translatable mirror at the equal path length position ($\tau = 0$).

2. Place a pointed object at the input of the instrument and look into the output of the instrument. Rotate the fixed mirror until the two images coincide.

3. With light from the sodium lamp incident upon the instrument, look into the output of the instrument. You should see fringes. If you don't, repeat (l) and (2). Rotate the fixed mirror to form circular fringes, or at least strong fringes. Now place a white card at the output to view the fringes.

When you have centered the circular fringes, visually check the alignment of the lamp, interference filter, and interferometer. A crooked light path can cause subtle problems later. When you have centered the fringes on the photodiode, translate the mirror by turning the micrometer drive and note qualitatively the periodic appearance and disappearance of the fringes. Be careful not to go too far from the $\tau = 0$ position because the nonzero width of the spectral lines comprising the sodium doublet leads to an eventual washout of the fringes ($V(\tau) \rightarrow 0$ as τ becomes large). While viewing the fringes by eye, you should measure roughly the distance from one visibility minimum to the next. Compare your measured value with the value you calculate using the literature value for the spacing of the sodium doublet. If the values agree, you know what you're looking at!

The goal of Part I is to measure the doublet separation $\Delta\omega$ with a precision better than 1%. Many students are able to achieve this goal simply by observing visually several successive visibility minima (up to 6 or more) while recording the micrometer readings at the minima. A plot of the micrometer readings versus minimum number (simply an integer) should be a straight line. A least-squares fit should yield a slope (with uncertainty) from which the doublet splitting can be deduced. If you use this technique, be sure to use a sample variance technique to assign uncertainties to the positions of the minima.

Alternatively, you can measure the visibility using the photodiode. <u>Lightly</u> touch the top of the mirror translation dial or <u>gently</u> press on the optical rail or table that support the interferometer. A <u>light</u> touch or <u>gentle</u> press will cause the fringe pattern to shift by a few fringes, which in turn causes the output intensity $I(\tau)$ measured by the photodiode to vary through a few cycles of the rapid fluctuation term $\cos\,\omega_{ave}\tau$. If the digital oscilloscope is in its "STORE" mode, and the horizontal time scale is set to a few milliseconds per division, then the fluctuating output of the photodiode amplifier will result in a band of brightness on the scope screen. The bottom and top of the bright band correspond to the minimum and maximum values of $I(\tau)$ for τ in an interval small enough for the visibility $V(\tau)$ to be constant. Hence from the vertical height of the bright band and the vertical position of its midpoint, a value of $V(\tau)$ may be calculated. Be sure to cover the photodiode with your hand and measure its output level with no incident light – this quantity, if significant, must be accounted for in your calculation of $V(\tau)$.

By advancing the micrometer, repeating the "light touch" or "gentle press" maneuver, and recording again the band of brightness, values for the visibility may be obtained at chosen values of τ. An interpolation technique can be used to find the positions of successive minima. Again, be sure to use a sample variance technique to determine an uncertainty in the distance between successive minima, and propagate this uncertainty through to an uncertainty in the doublet separation.

Convert your measured value for the sodium doublet separation $\Delta\omega$ into a difference in wavelength (assume the average wavelength is 589.3 nm). Compare your measured value with the literature value - it should be roughly 0.6 nm (or 6 angstroms). Note that 1% of 0.6 nm is 0.006 nm. Take a moment to muse on the magnitude of 0.006 nm.

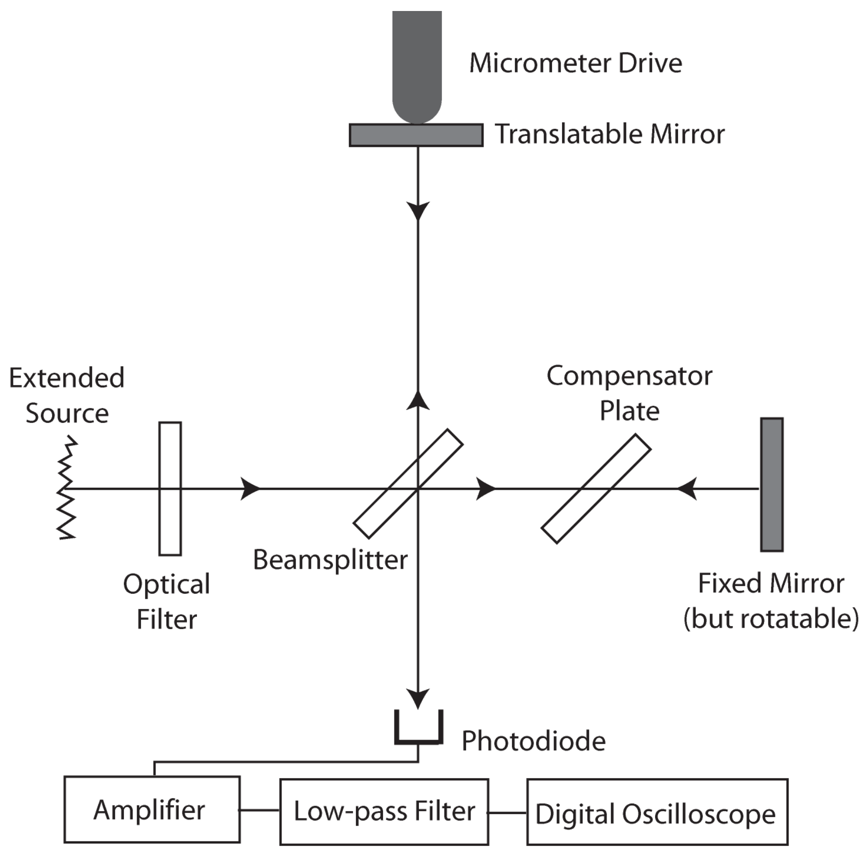


Figure 5-2. Experimental setup for Parts I and II.

Part II - Measurement of the width of the mercury green line.

Replace the sodium discharge lamp with the mercury discharge lamp. Be sure to replace the sodium doublet interference filter with the interference filter for the 546 nm mercury green line. After centering the fringes on the photodiode, translate the mirror of the spectrometer and notice the decrease in the visibility of the fringes as the mirror is translated in either direction from the $\tau = 0$ position. The photodiode technique (as described in Part I) will be particularly helpful here even if you did not use it in Part I. Remember, though, that the width of the bright band in this technique tells you only ($I_{max} - I_{min}$), and you will need to know the overall offset from zero in order to calculate and plot the fringe visibility.

You should measure the visibility of the fringes as a function of $\tau = 2d/c$. Be sure to take data in both directions from the $\tau = 0$ position, and check to see that the data is symmetric about $\tau = 0$.

For a Lorentzian spectral line, a plot of $\ln[V(\tau)]$ versus τ should be linear with a slope of ($-1/\tau_p$). Using a linear regression technique, calculate a value and an uncertainty for τ_p, the mean time between collisions of mercury atoms. Your uncertainty in τ_p should be the result of sample uncertainties in measured parameters. Convert ($1/\tau_p$), the HWHM of the spectral line (in radians/sec) to a width in nm and compare this width with the center wavelength of 546 nm. If you have performed the grating spectrometer experiment, compare your two values for the width of this line. If you haven't performed the grating spectrometer experiment, indicate the necessary resolving power of a grating which could detect the nonzero width of this line. Are such gratings available commercially?