Experiment 6 - Tests of Bell's Inequality

References:

"Entangled photon apparatus for the undergraduate laboratory," and "Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory," two papers by D. Dehlinger and M. W. Mitchell, *American Journal of Physics* **70** (9) 898-902 and 903-910 (2002). These papers focus on tests of Bell's inequalities using very much the same experimental setup as our own. Hardcopies of these papers are provided in a 3-ring binder in the lab.

"Quantum mysteries tested: An experiment implementing Hardy's test of local realism," by J. A. Carlson, M. D. Olmstead, and M. Beck, *Am. J. Phys.* **74** (3) 180-186 (2006). Excellent guide for performing Hardy's test with our experimental setup. A hardcopy is provided in a 3-ring binder in the lab.

I. Introduction

Recent work in Advanced Lab and Optics Lab has moved beyond tests of the quantum nature of light and has focused on tests of Bell's inequality which compares the predictions of hidden variable theories with those of quantum mechanics. The papers by Dehlinger and Mitchell (*Am. J. Phys.* 2002), especially the second one (pp. 903-910), are excellent guides to these tests. With our experimental setup, tests of Bell's inequality are implemented by exploiting the polarization state of the entangled photons produced in the spontaneous parametric down-conversion (SPDC) process. Initial work in Advanced Lab and Optics Lab employed Hardy's test of local realism as described in the paper by Mark Beck's group (*Am. J. Phys.* 2006). Hardy's test is generally considered the test of Bell's theorem which is easiest to understand. However, the entangled photon states produced in our experimental setup (and in most similar setups) are only "80% pure", and this fact led to many failed attempts to use Hardy's test to demonstrate the superior predictive capabilities of quantum mechanics over hidden variable theories. During the Fall of 2009, David Berryrieser and Rob Warren (HMC '10) showed that Hardy's Test could be made to work, but eventually transitioned to the more general approach of Dehlinger and Mitchell. They successfully demonstrated that quantum mechanics describes nature accurately in situations where hidden variable theories cannot. Several workers in Optics Lab have since repeated and extended the results of Berryrieser and Warren. Recently we acquired a "pre-compensation" quartz crystal which should provide 95% pure entangled photon states and a substantial improvement in our ability to rule out hidden variable theories as explanations for our measurements. Initial attempts to use the crystal have not improved the entanglement purity as much as expected (see Anthony Corso's Optics Lab tech report, spring 2014); solving this puzzle is a worthy goal for this semester!

II. The Experimental Setup

The experimental setup for performing tests of Bell's inequality is sketched in Fig. 4-1 (also see photo in Fig. 4-2). The entangled photon-pair source is comprised of a 50 mW violet laser (405 nm) illuminating a pair of beta-barium borate (BBO) crystals cut to facilitate type-I spontaneous parametric down-conversion (SPDC). In this non-linear process an occasional incident 405 nm photon is converted into a pair of 810 nm photons with linear polarization orthogonal to the linear polarization of the incident beam. The 405 nm half-wave plate provides a means for rotating the polarization of the incident beam so that roughly equal numbers of horizontally and vertically polarized 810 nm photon pairs are produced in the SPDC process. Conservation of momentum and energy imposes constraints on the two 810 nm photons, so that they are entangled in energy, momentum, and polarization. An 810 nm photon in the bottom path of Fig. 4-1 is often called an “idler” photon and is used as a gate for coincidence circuitry, while an 810 nm photon in the top path is called the “signal” photon. In our work in testing Bell’s inequality, we may refer to idler photons as “gate” photons, and the signal photons may also be called “transmit” photons. The “gate” and “transmit” nomenclature is a remnant of using this setup for the test of the quantum nature of light. In actual fact, these two paths are on an equal footing in tests of Bell’s inequality, and we simply take advantage of the coincidence circuitry to measure when an entangled photon pair is transmitted successfully through the two Glan-Thompson polarizers.

The pair of BBO crystals in Fig. 4-1 consists of two 0.5 mm-thick crystals rotated so that their crystal axes are effectively perpendicular, and then cemented together. The result is that a horizontally-polarized 405-nm photon incident upon one crystal can generate a pair of entangled 810 nm photons with their polarization vertical, while a vertically-polarized 405 nm photon incident upon the other crystal can generate a pair of

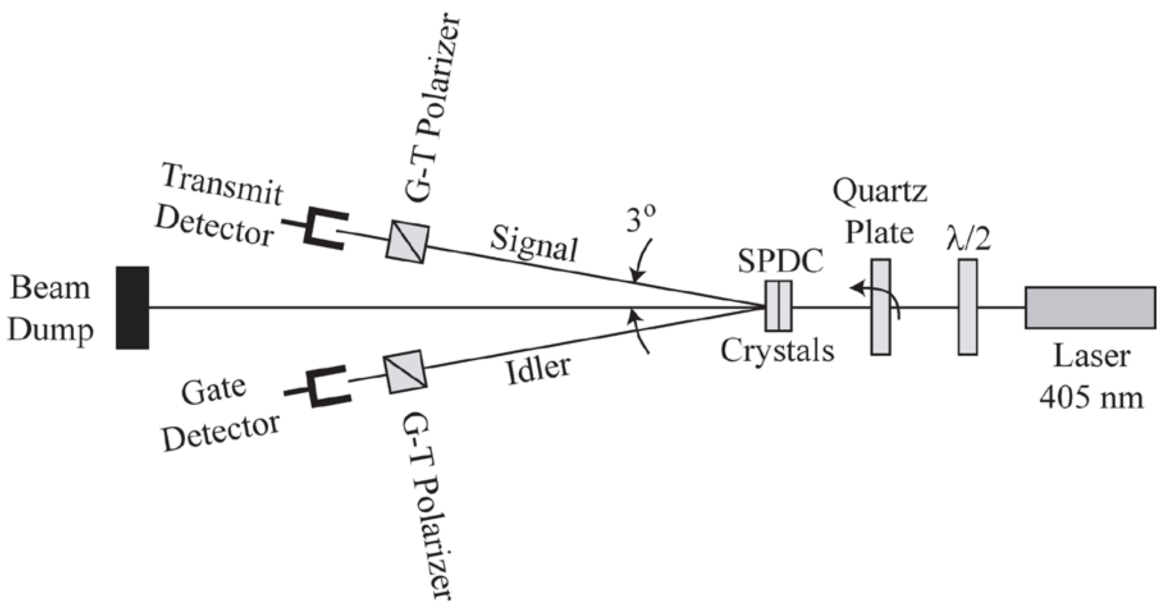

Fig. 4-1. Top view of the experimental setup for performing tests of Bell’s inequalities. SPDC = spontaneous parametric down-conversion.

G-T = Glan-Thompson. The quartz plate can be rotated about a vertical axis.

horizontally-polarized photons. A couple of meters down-stream of the BBO crystals, where the polarizers and detectors are located, it is impossible to tell where the photon pair was created because, even in principle, there is insufficient depth resolution looking back at the BBO crystals to place the origin of the photon pair in one crystal or the other. Hence the photons are entangled with respect to polarization as well as momentum and energy. The 405-nm laser emits horizontally-polarized photons, so if the optic axis of the half-wave plate is oriented vertically (0°) (or for that matter, horizontally (90°)), a pair of vertically-polarized photons is generated. Hence the maximum coincidence count rate should occur, in this case, with the polarizers oriented vertically (0°).

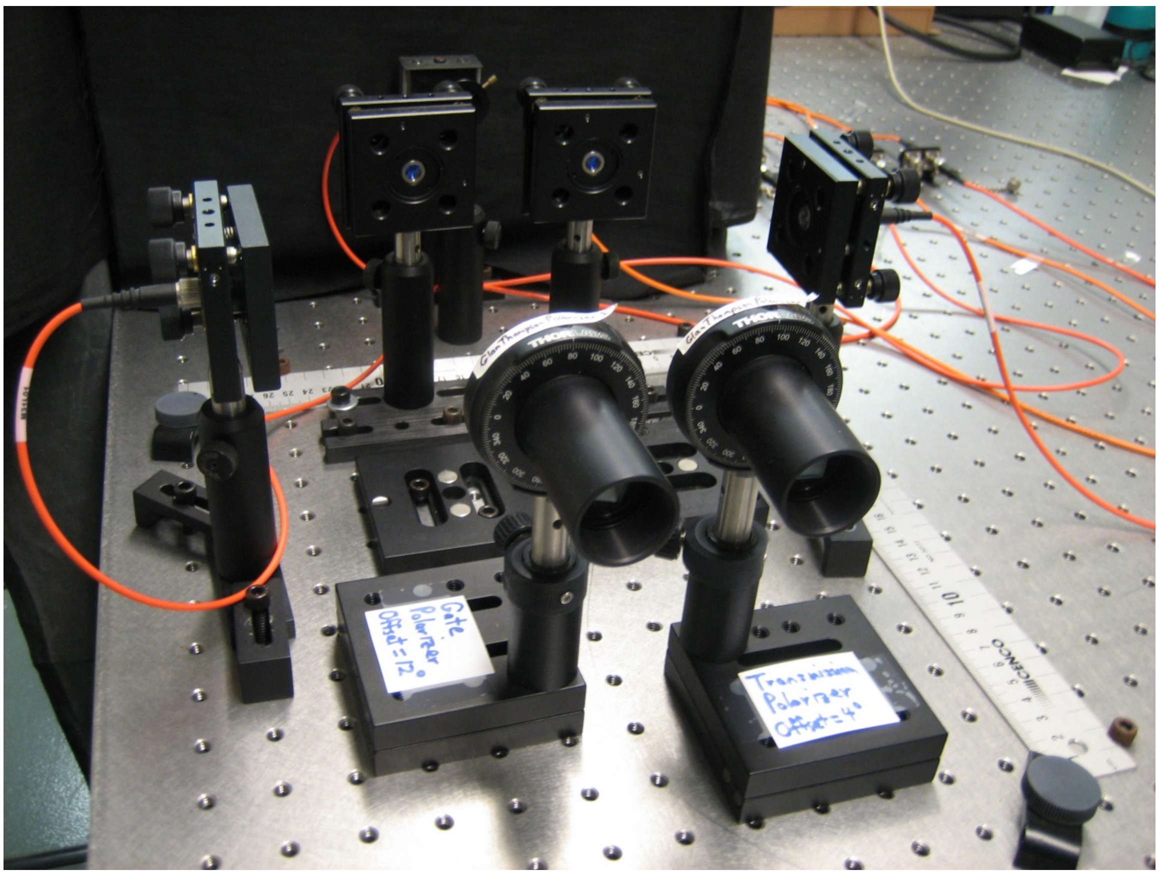

Fig. 4-2 A photo of the polarizers and fiber couplers in the Bell inequality setup. The 405 nm laser, quartz plate, 405 nm half-wave plate, and BBO crystals (all not shown) generate 810 nm entangled photon pairs that propagate through the idler (gate) polarizer (shown on the left) and the signal (transmission) polarizer (on the right) and head toward their respective fiber couplers (face-on at the top of the photo). The 810 nm photons are carried by optical fiber to the silicon avalanche photon-counting detectors housed in the black box shown at the top of the photo.

The birefringent quartz plate in Fig. 4-1 provides a means of zeroing the phase ϕ in the QM state for the entangled photon pairs:

$$\left|\psi_{DC}\right\rangle = a\left|H\right\rangle_s\left|H\right\rangle_i + b\exp(i\phi)\left|V\right\rangle_s\left|V\right\rangle_i \tag{4.1}$$

where normalization requires $|a|^2 + |b|^2 = 1$, and $|a|^2$ is the probability that the photon pair is polarized horizontally and $|b|^2$ is the probability of vertical polarization. The optic axis of the quartz plate is oriented vertically, so a horizontally polarized photon pair will experience a different refractive index than does a vertically polarized photon pair, and hence a phase difference is introduced between the two polarizations. Rotating the quartz plate about a vertical axis changes the effective thickness presented to the 405 nm beam, thus varying the phase difference imparted to the two polarization states. This is the method used to set $\phi = 0$ in Eqn. (4.1).

For a more detailed description of the apparatus, see the papers by Dehlinger and Mitchell and by Mark Beck's group at Whitman College (hardcopies are available in the 3-ring binders in the lab).

III. Performing Tests of Bell's Inequality

The successful completion of a test of a Bell inequality involves mastery of a number of techniques: the quantum mechanical calculations involved in predicting count rates, the calibration of the two Glan-Thompson polarizers, a thorough understanding of the coincidence electronics and data acquisition equipment, and a working familiarity with the operation of the half-wave plate and the quartz plate. The goal of our work is to perform and document successful measurements that conclude that hidden variable theories cannot explain our experimental results while the standard theory of quantum mechanics can!

Dehlinger and Mitchell (*Am. J. Phys.* **70** (9) 903-910 (2002)) describe in detail the procedure for performing a test of a Bell inequality first derived by Clauser, Horne, Shimony, and Holt, the so-called CHSH Bell inequality. You will need to read this paper carefully so that you understand the theory thoroughly and appreciate the motivation behind every step in their experimental procedure. A few words of advice should prove helpful and are included below.

You may want to begin by checking the orientation of the transmission axes of the signal (transmit) and idler (gate) Glan-Thompson polarizers. In the Fall of 2009, Alex Steinkamp (HMC '10) used a polarizing beam splitter to determine the rotation stage settings of the signal and gate polarizers that correspond to transmission of vertically polarized light. Alex and subsequent collaborators found that the signal (transmission) stage reads 4° while the idler (gate) reads 12°. The expected values are 0°, so the idler reading is hard to believe, but it seems to be true. A simple method can be used to check on the *relative* settings of the two polarizers. With the optic axis of the 405 nm half-wave plate set vertically (actual rotation stage reading is –1.5°), and with the signal (transmit) polarizer set at 4° (its vertical setting), rotate the idler (gate) polarizer while recording the signal-idler (GT) coincidence count rate. The maximum coincidence count rate should occur at 12°. Then with both polarizers set for vertical transmission, rotate the half-wave plate to check on the reading (–1.5°?) that gives the maximum coincidence count rate.

Now rotate the half-wave plate to achieve linear polarization at 45° to the vertical. (This should be the case when the half-wave plate rotation stage reads 21°?) Check to see that the coincidence counts *N(0°,0°)* and *N(90°,90°)* are equal. Note that Dehlinger and Mitchell define $N(\alpha,\beta)$ to be the coincidence counts when the signal (transmit) polarizer is rotated to α (taking into account any offset in the rotation stage of the polarizer) and the idler (gate) polarizer is rotated to β.

Next tune the QM state to achieve $\phi = 0$ in Eqn. (4-1), resulting in the Einstein-Podolsky-Rosen state:

$$|\psi_{EPR}\rangle = \frac{1}{\sqrt{2}}\left[|H\rangle_s|H\rangle_i + |V\rangle_s|V\rangle_i\right] \qquad (4\text{-}2)$$

This is accomplished by rotating the quartz plate until the measured coincidence counts *N(45°,45°)* are maximized (about 30°?). You should perform the calculations to show that $\phi = 0$ indeed corresponds to a maximum in *N(45°,45°)*. As a double-check, you might want to confirm both theoretically and experimentally that *N(–45°,45°)* is a minimum when $\phi = 0$.

Finally, plan the sixteen measurements of coincidence counts that will allow you to evaluate S defined by (see Dehlinger and Mitchell):

$$S \equiv E(a,b) - E(a,b') + E(a',b) + E(a',b') \qquad (4\text{-}3)$$

where $$E(\alpha,\beta) \equiv \frac{N(\alpha,\beta)+N(\alpha_\perp,\beta_\perp)-N(\alpha,\beta_\perp)-N(\alpha_\perp,\beta)}{N(\alpha,\beta)+N(\alpha_\perp,\beta_\perp)+N(\alpha,\beta_\perp)+N(\alpha_\perp,\beta)} \qquad (4\text{-}4)$$

and $$a = -45°, a' = 0°, b = -22.5°, b' = 22.5° \qquad (4\text{-}5)$$

(note the misprint in Dehlinger and Mitchell, page 907, in which they erroneously say $b = 22.5°$ instead of $b = -22.5°$)

and, for example, $$\alpha_\perp = \alpha + 90° \qquad (4\text{-}6)$$

You will probably find it useful to construct a table with all of the pairs of angles that you want to employ, taking account of the offsets in the readings of the rotation stages of the polarizers. Then methodically go through the sixteen measurements. Be sure to use a sample variance approach to determining your uncertainty – don't assume Poisson statistics as Dehlinger and Mitchell do, though that's probably not a bad assumption, just unnecessary!

According to the CHSH Bell inequality, $S \leq 2$, so if your calculated (measured) value of S is greater than 2 with statistical significance, then you have shown that hidden variable theories cannot accurately describe the measurement you have performed, but presumably quantum mechanics can! The less-than-perfect purity of the state may lead to a value of $S < 2\sqrt{2} \approx 2.83$ which is the predicted value for a perfect EPR quantum mechanical state (see Eqn. (4-2)).