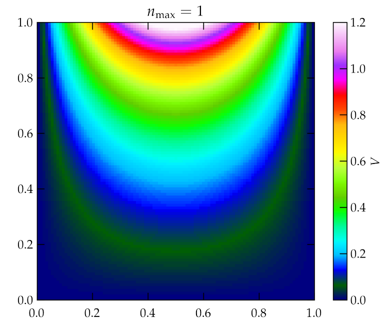
Problem Set 6

due: Monday, 2 March 2026

Physics 64



Problem 1 – Differentiable? Show that the following functions are differentiable functions of $z = x + iy$ ($x, y \in \mathbb{R}$):

(a) e^{ikz}

(b) $1/z$

(c) $\sqrt{z}$

Problem 2 – Sanity Check Note that

$$\frac{1}{1+x^2} = \frac{i}{2}\left[\frac{1}{x+i} - \frac{1}{x-i}\right]$$

By integrating the right-hand side, show that

$$\int_0^1 \frac{\mathrm{d}x}{1+x^2} = \frac{\pi}{4}$$

Problem 3 – A complex series Show that

$$\sum_{n=1}^{\infty} \frac{i^n}{n} = i\frac{\pi}{4} - \frac{1}{2}\ln 2$$

Hint: one clever way to sum a series such as $S = 1 - \frac{1}{3} + \frac{1}{5} - \cdots$ is to define

$$f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$$

and note that $f(1)$ is the desired sum S. On differentiating $f(x)$, we get

$$f'(x) = 1 - x^2 + x^4 - \cdots$$

which is a geometric series that you can sum in closed form. On integrating,

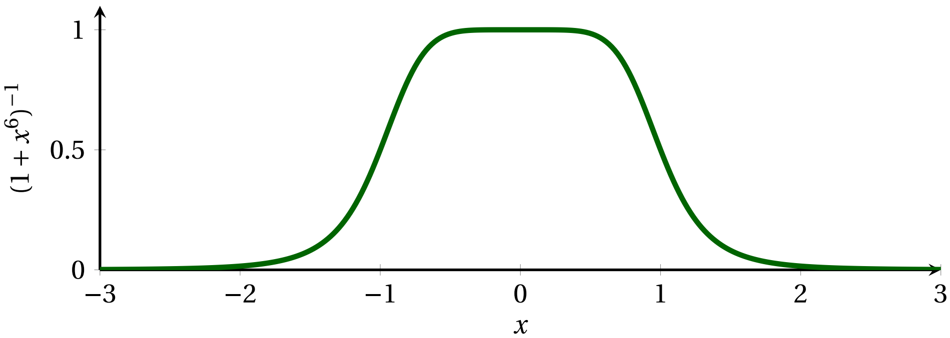
$$\int_a^1 f'(x)\,\mathrm{d}x = f(1) - f(a)$$

for a skillfully chosen a, you can show that $S = \frac{\pi}{4}$.

Problem 4 – A Curious Integral Use the calculus of residues to evaluate the integral

$$I = \int_{-\infty}^{\infty} \frac{1}{1+x^6}\,\mathrm{d}x$$

for which the integrand is shown below.



Check your answer by using scipy.integrate.quad to perform the integral numerically.