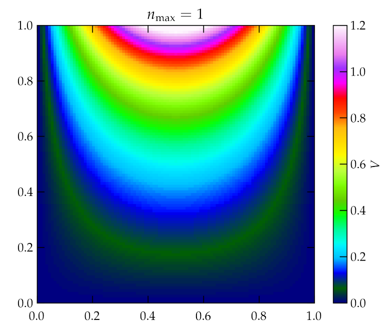

Physics 64

Problem Set 7

due: Monday, 9 March 2026

Problem 1 – General properties of Fourier transforms and convolutions Assuming that the Fourier transform of $f(t)$ is $\tilde{f}(\omega)$,

(a) show that if $f(t)$ is real, then $\tilde{f}(-\omega) = [\tilde{f}(\omega)]^*$.

(b) show that if $f(t)$ is even or odd, then so is its Fourier transform.

(c) if $f(t)$ is imaginary and odd, what can you deduce about its Fourier transform?

(d) show that $f * g = g * f$, meaning that the convolution of f and g is commutative.

Problem 2 – Fourier transform of a triangle Consider the Fourier transform $\tilde{f}(k)$ of the function

$$f(x) = \begin{cases} A(a - |x|) & -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

(a) Should $\tilde{f}(k)$ be even or odd? real or imaginary?

(b) Compute $\tilde{f}(k)$.

(c) Explain why the way $\tilde{f}(k)$ changes with increasing a makes sense. Hint: you may find it useful to plot $\tilde{f}(k)$.

Problem 3 – Fourier transform of a SHO eigenfunction Calculate the Fourier transform of the first excited state of a simple harmonic oscillator, which has the form

$$\psi(x) = Axe^{-\alpha x^2}$$

where A is a normalization constant and α is a positive constant. Does it have the same property as the ground state—that the functional form of the Fourier transform is the same as the function itself?

Problem 4 – Calling Hendrik and Augustin-Louis Calculate the Fourier transform of

$$f(x) = Ae^{-\alpha|x|}$$

where A is a normalization constant and α is a positive constant.