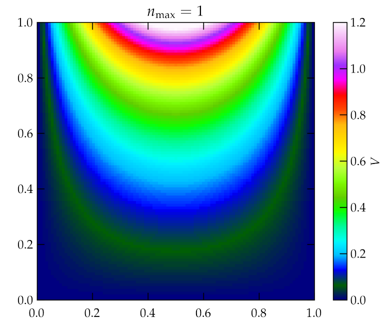
Problem Set 10

due: Monday, 13 April 2026

Physics 64



Problem 1 – Euler's (Equidimensional) Equation The ordinary differential equation

$$x^2 y''(x) + (2\alpha + 1) x y'(x) + \beta y(x) = 0 \tag{1}$$

for dimensionless constants α and β, is **equidimensional**, since each term has whatever dimensions y has.

(a) Look for a solution of the form $y = x^r$. What is the quadratic equation that r must solve? (This is called the **indicial equation**.)

(b) What are the two linearly independent solutions if $\alpha = 1$ and $\beta = -1$?

(c) What is the condition for there to be a single value of r?

(d) When there is only one value for r, the second linearly independent solution is not a simple power of x. Instead, show that $y = x^r \ln x$ is a second solution to the differential equation.

Problem 2 – Make up your mind! (after Nearing 10.12) A thick slab of material of density ρ and specific heat capacity c is alternately heated and cooled at its top surface ($x = 0$) so that the surface temperature oscillates between T_1 and $-T_1$ with period τ according to

$$T(0,t) = \begin{cases} T_1 & 0 < t \bmod \tau < \tau/2 \\ -T_1 & \tau/2 < t \bmod \tau < \tau \end{cases}$$

Find the temperature inside the material for $x > 0$ (x is oriented positive inside the material away from the top surface).

Problem 3 – Potential inside a cube Consider a cube of side L with conducting walls. One corner of the cube is at the origin; the other is at (L, L, L). All sides of the cube are grounded, except the top plate at $z = L$, which is held at potential V_0. Noting that the potential inside the cube satisfies Laplace's equation,

$$\nabla^2 V = 0$$

solve for the potential $V(\mathbf{r})$ inside the cube.