
Fraunhofer and Fresnel Diffraction

Diffraction Patterns of a Circular Aperture and a Straight Edge

References

- *Optics* by Eugene Hecht, Chapter 10
- *Introduction to Modern Optics* by Grant Fowles, Chapter 5
- *Principles of Optics* by Max Born and Emil Wolf, Chapter 8
- The following reference contains a superb treatment of Gaussian spherical beams: *An Introduction to Lasers and Masers* by A.E. Siegman, Chapter 8

1.1 Introduction

This experiment will extend your knowledge of Fraunhofer diffraction and introduce you to situations in which the more general Fresnel diffraction calculations are required. In Section I you will measure the Fraunhofer diffraction pattern produced by a circular aperture and compare your data with the predictions of scalar diffraction theory. In Section II you will spatially filter a helium-neon laser beam and study the famous Fresnel diffraction pattern of a straight edge.

In both parts of the experiment, you will want to begin by simply looking at the diffraction pattern you are attempting to measure. Then become familiar with the measurement capabilities and limitations of the experimental setup. With this knowledge you can then think and plan carefully to obtain the best possible measurements of your diffraction patterns in a reasonable amount of time.

Once you have taken your data appropriately (in Section I or in Section II), this experiment affords an excellent opportunity to witness the power of error analysis and nonlinear χ^2 fitting.

Take your results seriously and expect to obtain good fits with reasonable χ^2 values and random residuals. If you don't achieve them, you should investigate and understand any effects which keep you from getting them!

1.2 Fraunhofer Diffraction Pattern of a Circular Aperture

You should read carefully an account of the calculation of the Fraunhofer diffraction pattern produced by a plane wave incident upon a circular aperture. Most introductory optics texts include this calculation, and *you should write your own version in your notebook*. The references listed above have good treatments of scalar diffraction theory, although you are encouraged to consult any of the other optics texts available to you.

We would like to test the prediction that the intensity in the diffraction pattern is given by

$$I(\theta) = 4I_{\max} \frac{[J_1(ka \sin\theta)]^2}{(ka \sin\theta)^2} \quad (1.1)$$

where θ is the angle of diffraction, $k = 2\pi/\lambda$, a is the **radius** of the aperture, and J_1 is the Bessel function of first order. The experimental setup includes a 5-mW helium-neon laser (JDS Uniphase Model 1125P, $\lambda = 632.8$ nm), a circular aperture with a *diameter* of roughly 200 μm , and a photodiode detector with amplifier. **Do not touch the circular aperture; finger grease and dead skin cells severely degrade the quality of the diffraction pattern.** With this equipment you can record the intensity at a number of points in the diffraction pattern, and then fit the data to the functional form of Eq. (1.1). Appendix A at the end of the lab manual describes how to perform the nonlinear least squares fit to the circular aperture data using Igor Pro.

Start by directing the laser beam onto the 200- μm -diameter aperture. (See Fig. 1.1.) The diffraction aperture is mounted on an optical rail carrier with horizontal and vertical translation, so the aperture can be centered in the laser beam. At the output of the laser, the beam forms a waist (Gaussian spherical beam waist) with a spot diameter of $2w_0 = 0.83$ mm. In other words, the surface of constant phase of the electric field is a plane, and a circular aperture with a diameter of 0.83 mm will pass 86% of the beam power. The intensity of the beam falls off in the transverse direction (i.e., perpendicular to the beam propagation) as $\exp(-2r^2/w_0^2)$ where r is the distance from the beam axis. The finite diameter of the beam effectively forms an aperture through which the beam is continuously diffracting, so the beam is often described as “diffraction-limited.” This process of diffraction leads to divergence of the beam as it propagates away from the laser output (in our case, along the optical rail); the beam divergence half-angle is $\theta_{1/2} = \lambda/\pi w_0 = 0.05$ mrad. Most treatments of Fraunhofer diffraction from a circular aperture assume a constant intensity over the aperture. **You will have to decide what distance s you want between the laser and the diffraction aperture so that the intensity over the area of the aperture is roughly constant.**

There are other considerations in arranging the experimental setup. The distance s from laser to diffraction aperture and the distance r from diffraction aperture to the photodiode detector

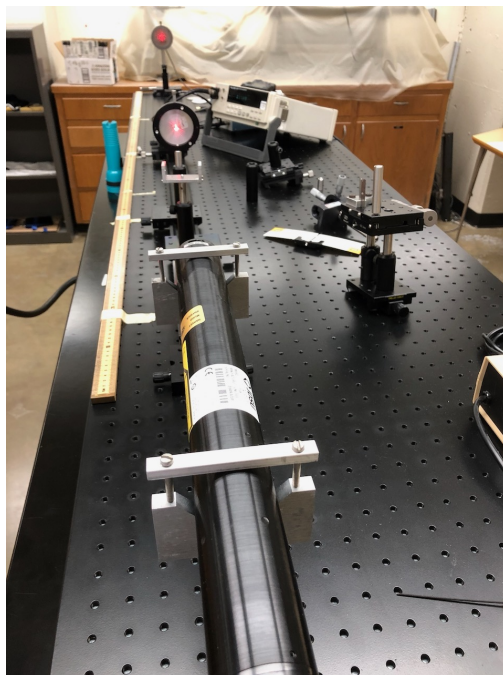


Figure 1.1: The experimental setup for measuring the Fraunhofer diffraction pattern of a circular aperture. The HeNe laser is at the bottom and its red beam impinges upon the 200- μm -diameter circular aperture to the right of the green flashlight. The diffraction pattern appears as a red blob on the photodetector at the top of the image.

should be chosen so that the following *Fraunhofer condition* is satisfied:

$$\frac{s}{d} \gg \frac{d}{\lambda} \quad \text{and} \quad \frac{r}{d} \gg \frac{d}{\lambda} \quad (1.2)$$

where $d = 2a$ is the diameter of the diffraction aperture. When this condition is satisfied (a ratio of 10 should do) the diffraction pattern formed on the detector aperture is the Fraunhofer pattern. The detector aperture (the aperture immediately in front of the photodiode) must be chosen judiciously. Larger apertures allow more light to hit the photodiode and hence give more signal, but they average over a larger area of the diffraction pattern. (Note that since the active area of the photodiode is roughly 2.5 mm in diameter, there is no point in using a detector aperture with a diameter larger than 2.5 mm.)

You should choose a detector aperture which is the optimal compromise between too little signal and too much spatial averaging. Of course, increasing the distance r between the diffraction aperture and photodiode also reduces the effect of spatial averaging and decreases the overall intensity on the photodiode.

Record the intensity at an appropriate number of points in the diffraction pattern. Uncertainties in the data points should be determined by forming a sample variance of several scans through the pattern.

The dependence of $I(\theta)$ in Eq. (1.1) on the fitting parameter a is certainly not linear. Bevington and Robinson §8.6 describe a computer program MARQUARDT which performs least squares fits to fitting functions which are nonlinear functions of fitting parameters. MARQUARDT is based on an algorithm published in 1963 by D. W. Marquardt (*J. Soc. Ind. Appl. Math.* **2**: 431–441 (1963)). This same algorithm, often called the Levenberg-Marquardt algorithm, is the basis of the nonlinear least squares fitting routines in the commercially available software packages, such as Igor Pro, Origin, or MATLAB. The Appendices at the end of the lab manual contain tutorials for these software packages.

The instructions in the Appendices will lead you step-by-step through the mechanics of fitting your data. The main challenge for you will be to apply your understanding of χ^2 fitting and error analysis in order to guide the fitting routine most efficiently and interpret its output sensibly.

Don't be upset if your value for the radius a of the aperture deviates from $100\ \mu\text{m}$. This value is a nominal value specified by the manufacturer with a tolerance of 10%. Concentrate on obtaining a good, self-consistent fit. *Aim to attempt a fit to your data while still in the laboratory, so that you can seek help from your instructor if you encounter difficulties using Igor Pro.*

1.3 Fresnel Diffraction by a Straight Edge

Read carefully an account of the calculation of the Fresnel diffraction pattern of a straight edge. Most optics texts treat this classic example; many students find the treatment in the reference by Fowles particularly clear. Astrophysicists measure the Fresnel diffraction pattern produced on the earth's surface by starlight as it passes the edge of the moon's disk (called lunar occultation). Deviations from the expected pattern for a point source enable them to deduce a value for the diameter of the star. The disk of the star is otherwise unresolvable by earth-bound telescopes.

You can create a point source with a substantial cone of light by spatially filtering the laser beam. The spatial filter consists of a microscope objective and a circular aperture, housed together. The microscope objective produces a focused waist of the laser beam shortly beyond the objective. The aperture transmits only the waist of the Gaussian spherical laser beam, and blocks stray reflections and imperfections in the beam. The final result is a beautiful cone of light with no peripheral imperfections — hence the name “spatial filter”. Chapter 8 of the reference by Siegman contains an excellent description of Gaussian spherical beams.

Mount the spatial filter on the optical rail carrier with horizontal and vertical translation. (See Fig. 1.2.) Remove the $25\text{-}\mu\text{m}$ -diameter aperture from the spatial filter (after noting its orientation) and direct the laser beam through the microscope objective, using the horizontal and vertical translation of the carrier to center the objective on the laser beam. Replace the $25\text{-}\mu\text{m}$ -diameter aperture and use the horizontal and vertical micrometers on the spatial filter to maximize the intensity transmitted by the aperture. You may need to use the axial screw adjustment on the spatial filter to translate the microscope objective slightly forwards or backwards along

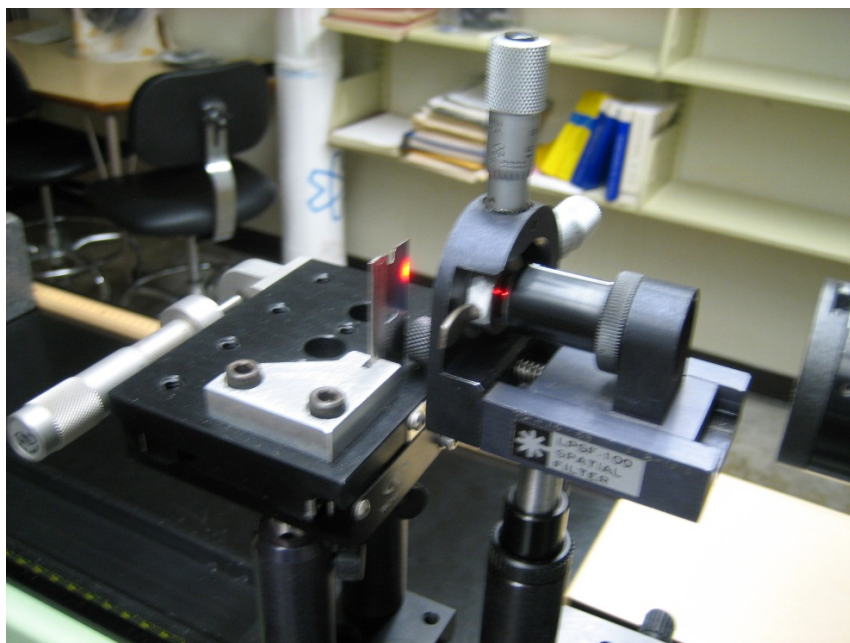


Figure 1.2: A spatial filter consists of a focusing lens (microscope objective) and a circular aperture. The laser beam is focused through the aperture while stray light from the laser resonator is caught by the edges of the aperture, leaving a very smooth and uniform beam incident upon the straight edge.

the beam path so that its focal plane coincides with the plane of the 25- μm aperture. When you have positioned the focused “waist” of the laser beam in the center of the 25- μm aperture, the transmitted intensity will be a maximum.

Direct the spatially filtered beam onto the razor blade, which is fastened to a micrometer-driven translator, which in turn is mounted on an optical rail carrier. Rather than translating the photodiode to scan the diffraction pattern, **you can simply translate the razor blade** using the micrometer. This procedure keeps the photodiode in the same portion of the spatially filtered beam. Use your ingenuity to devise a convenient geometry of spatial filter, razor blade, and detector. Compare your data with the predictions of Fresnel diffraction theory.

Some students in the past have become intrigued with the famous Poisson bright spot in the diffraction pattern of a ball bearing. In fact, if you point the spatially filtered beam at just about anything in the lab you’ll see some fairly remarkable wave phenomena. You are certainly free to test any of the predictions of diffraction theory in addition to—or instead of—the examples suggested above, but be sure to clear your plans with your instructor.

1.3.1 Recording 2-D Diffraction Patterns with a CCD Camera

During the summer of 2012, the Physics Department purchased a CCD camera from Thorlabs (DCU224M, 1280×1024 pixels, B&W, sensitive area $5.95 \text{ mm} \times 4.76 \text{ mm}$, pixel area $4.65 \mu\text{m} \times 4.65 \mu\text{m}$, frame rate 15 fps, purchase price \$2400). The camera communicates with a host computer via a USB 2.0 cable. This camera is available to us for recording 2-D diffraction patterns at speeds much, much faster than is possible by recording them point-by-point manually. And, of course, we can have a lot of fun fitting 2-D diffractions patterns or 1-D slices through these 2-D patterns with functions derived from Fraunhofer and Fresnel diffraction theory.

In Optics Lab during the spring of 2013, Peter Megson and Jaron Kent-Dobias (both HMC '14) used a red LED and crossed polarizers to demonstrate that the 1.3 megapixels of the DCU224M CCD camera respond linearly to incident light, and that the pixels are remarkably uniform across the entire array. (The tech reports of Peter and Jaron are available on the Sakai course site.) There is just one catch. A protective glass coverslip over the array results in an annoying interference pattern when a light source like a HeNe laser (with coherence length much longer than the thickness of the coverslip) is used to illuminate the camera. Jaron was able to reduce the effect of the interference pattern by a judicious normalization procedure, but we may want to explore ways to avoid the interference pattern altogether. For example, we could display a diffraction pattern of interest on a white screen and record the back-scattered light from the screen with the CCD camera. Or perhaps we could display the diffraction pattern of interest on a ground-glass screen and position the CCD camera to image the forward-scattered light transmitted through the ground-glass screen to the camera.

Actually, there is a second catch: the CCD camera's dynamic range is less than ideal for capturing in the same image the central maximum and the weaker peripheral rings in the diffraction pattern. You may need to develop a way to stitch together parts of the image with very different intensities.

There clearly is enormous potential for recording and fitting unusual 2-D diffraction patterns if we can successfully employ the CCD camera to record those patterns without artifacts. If you are interested in participating in the successful implementation of the camera to record and fit 2-D diffraction patterns, just chat with your instructor about how you can contribute to the mission! Work on the camera may replace some parts of Sections I and II if that seems appropriate, and there are certainly many opportunities here for ground-breaking technical report work!