Experiment 7

Geometrical Optics

You will be introduced to ray optics and image formation in this experiment. We will use the optical rail, lenses, and the camera body to quantify image formation and magnification; this will require you to become familiar with image capture software. At the end of the experiment you will design a microscope or telescope.

This experiment is under development and thus we appreciate any and all comments as we design an interesting and achievable set of goals.

Equipment

Optical rail; one-inch lenses; 40D camera body; light-blocking attachment to camera body; computer; translation stage on which to mount the camera; calipers (for magnification measurement); connecting cable (camera to computer); memory card; powder-free latex gloves; business cards to image; and image capture software.

7.1   Image Formation and Focal Length

The focal length of an ideal lens, f, is defined as the distance beyond the lens at which rays coming from infinity are brought to a point, as shown in the left diagram of Fig. 7.1. Note that the rays coming from infinity are parallel at the lens. Given Snel's law and a lens made of a dispersive material (as they all are), can you see why we expect rays of different wavelength (color) to have different focal lengths? This phenomenon is known as chromatic aberration and is shown in the right diagram of Fig. 7.1.
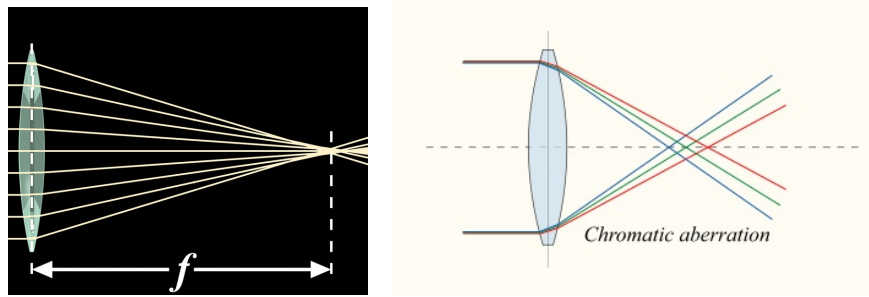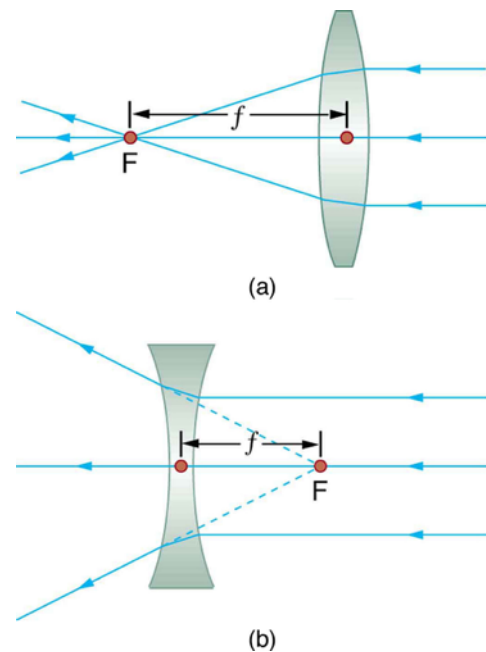
Figure 7.1: A lens focuses rays of light. When the rays are incident from very far away the lens forms an image at a distance from the lens equal to the focal length (left). Because the material from which a lens is made has slightly different values of the index of refraction for different colors; i.e., the lens is dispersive, the various colors of the incoming light come to focus at slightly different distances from the lens (right).

We can apply Snel's law to understand how lenses focus light, as shown in Fig. 7.2. Light is bent, or refracted, at both the front and the back surfaces of the lens, the front surface being the first surface the incident light encounters. Light incident at the front surface of the lens passes from a low n material (air) to a higher n material (glass) and, as described by Snell's law, is therefore bent toward the surface normal. Recall that θ is measured from the normal to the surface. Similarly, at the back surface of the lens, Snell's law dictates that the light is bent away from the normal. The combined effect of light entering and leaving the lens is that the light is focused.

While converging lenses, such as those shown in Fig. 7.1 have a positive focal length, diverging lenses have a negative focal length. If we trace these diverging rays backward we find that they come to a point on the same side of the lens as that from which the light rays are incident. Note that unlike the example of the converging lens, with a diverging lens the light rays never actually cross at a single point. Further, for variety, rays in Fig. 7.2 are shown incident from the right side of the lens not the left.

Using the example of the converging lens in Fig. 7.3 we can derive an expression that governs image formation in thin-lens systems in the limit of small angles. This is

Figure 7.2: A converging lens and a diverging lens. Ideal converging lenses focus incident parallel light to a point downstream of the lens at the focal length (a), while an ideal diverging lens defocuses that same light as though it came from a point upstream of the lens at the focal length (b).

often referred to as the "paraxial approximation". Recognizing similar triangles in Fig. 7.3 gives
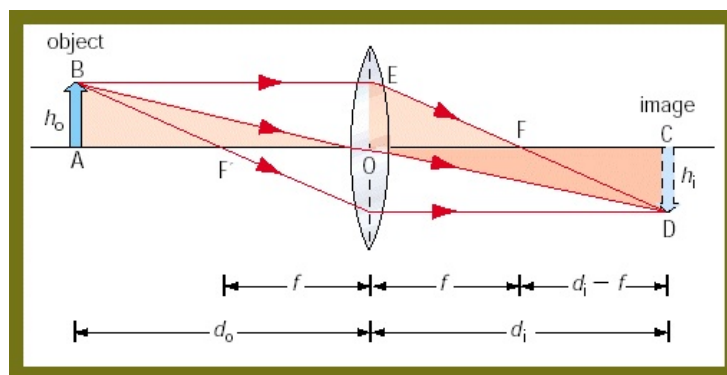
Figure 7.3: A single, thin, converging lens of focal length f forms an inverted image of an object.

$d_i/h_i = d_o/h_o$ and $h_i/(d_i - f) = h_o/f$, to arrive at the **thin-lens equation**:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$ (7.1)

The magnification, M, of a lens is given by

$$M = -\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$ (7.2)

where the minus sign indicates that the image is inverted. With a bit more effort, you can arrive at the **lens maker's formula**, which is applied to a thick lens whose front and back surfaces have radii of curvature R_1 and R_2, respectively:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{t(n-1)}{nR_1R_2}\right)$$ (7.3)

where n is the index of refraction of the lens material, t is the thickness of the lens as measured along the lens's center axis, R_1 is taken to be positive if the front surface (the first surface the light encounters) is convex, and R_2 is positive if the back surface is concave. The thin-lens approximation for this system would require that $t \ll |R_1|, |R_2|$, and under this assumption we would neglect the final term on the right side of this equation.

We can apply the thin lens equation to calculate the position of an image formed by a single lens, or multiple lenses. For instance, in the case of a two-lens imaging system, we start by using the thin lens equation to calculate the image position of the first lens, knowing the lens's focal length and the object distance. We then use this image position as the object position for the second lens. Knowing the object position and the focal length for the second lens, we can easily calculate the position of the image formed by the second lens. The magnification of the two lens system is the product of the magnifications of each lens, $M = M_1 M_2$. For a more complicated optical system with N lenses, we calculate the final image position and magnification as above, daisy-chaining the object position of the Nth lens to the image position of the $(N+1)^\text{st}$ lens.
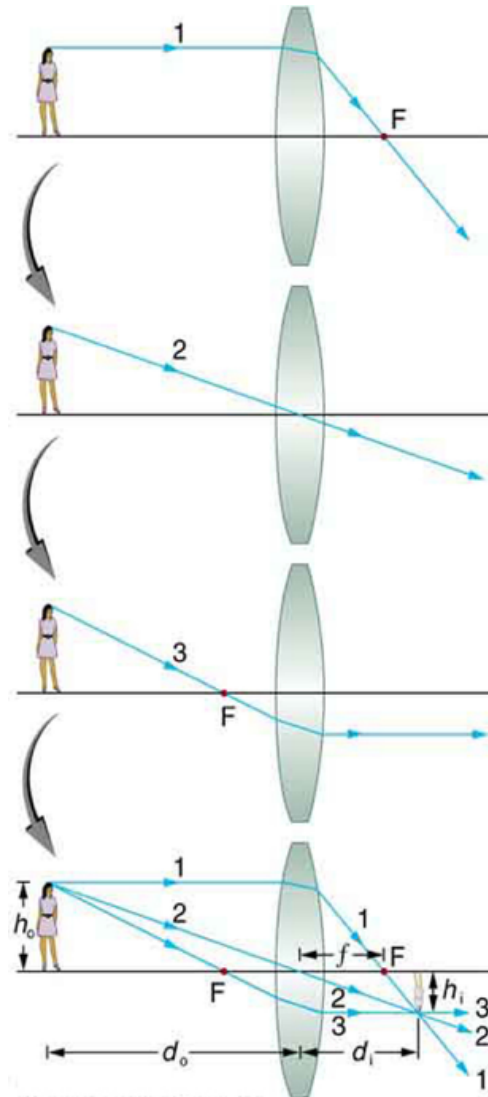
7.2 Image Formation and Ray Tracing for a Converging Lens

The thin-lens equation allows us to calculate the position at which an optical system will produce an image, in the thin-lens approximation. We can also calculate the image position using the ray method, which provides additional insight into how our optical system actually works. Using this method we find an image position by drawing three separate rays from our object through our thin lens. The rays are drawn:
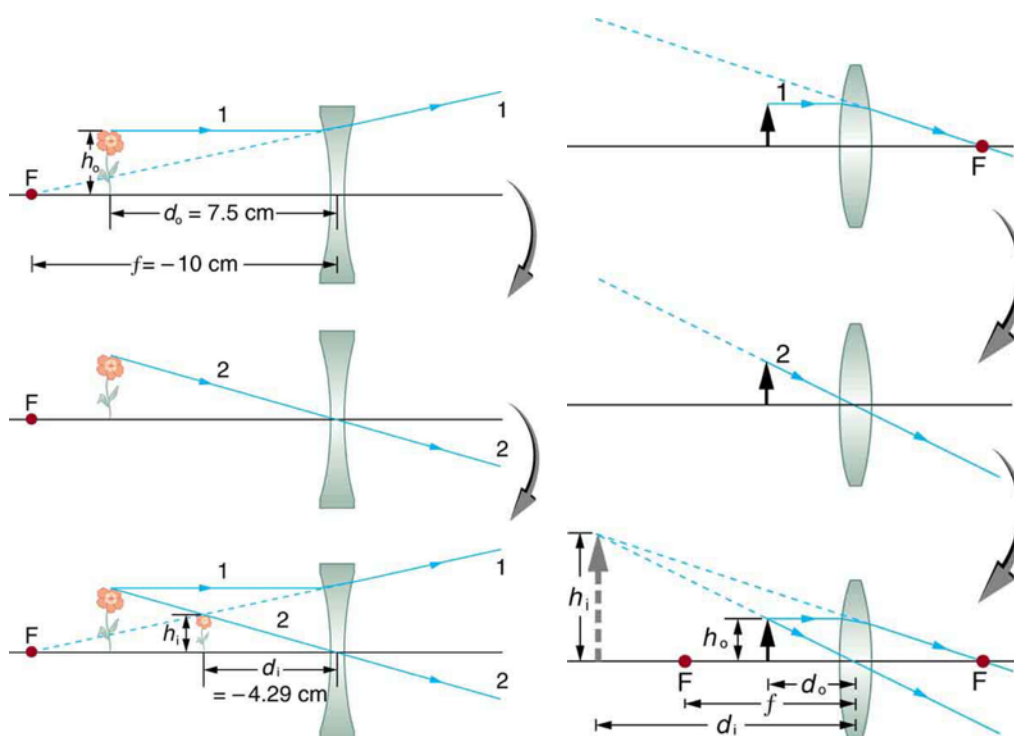
1. parallel to the lens axis, from the top of the object to the lens, and then through the focal point on the far side of the lens;

2. from the top of the object, undiverted through the center of the lens, to the far side;

3. from the top of the object, through the focal point on the near side of the lens, and then exiting the lens parallel to the lens axis on the far side.

The image forms along a plane perpendicular to the lens axis where the three rays cross. Figure 7.4 shows the ray method for a converging lens. If used precisely, the ray method is as accurate as the thin-lens equation; in fact, you might notice that we used these same three rays in Fig. 7.3 to derive the thin lens equation. Note that the light rays "downstream" from the image are indistinguishable from rays that would be produced by a source at the location of the image. This generalization of an image to be the location from which the rays originate will be useful in the discussion of virtual images and virtual objects.

Figure 7.4: A demonstration of the ray tracing method for a converging thin lens.

As mentioned above, diverging lenses have a negative focal length. This implies a negative image distance for any positive object distance. The image that we would form using the diverging lens is not a real image; it's what we call a virtual image—a mathematical construct that tells us where backward traveling rays would meet to form an image. This is equivalent to saying that the rays "downstream" from the lens are identical to rays that would be produced by a source of light at the position of the virtual image if the lens were not there.

Figure 7.5: Virtual images. On the left, a virtual image is formed using a diverging lens and, on the right, a virtual images is formed using a converging lens.

The light does not actually travel backward to form that image, so we call it virtual, and we could not detect that image with a sensor. It's worth noting that if you place the object at a distance less than the focal length of the lens, a converging lens will also produce a virtual image (see Fig. 7.5).

Figure 7.6 uses the ray method to illustrate how a camera or an eye forms an image.

Let's summarize some results of the preceding discussion:

- $f > 0, d_o > f$: d_i is positive (the image is real) and M is negative;

- $f > 0, d_o < f$: d_i is negative (the image is virtual) and $M > 1$;

- $f < 0$: d_o is positive, d_i is negative (the image is virtual) and $0 < M < 1$;

- If $|M| > 1$ the optical system magnifies; if $|M| < 1$ the optical system demagnifies; and if $M < 0$, the image is inverted.
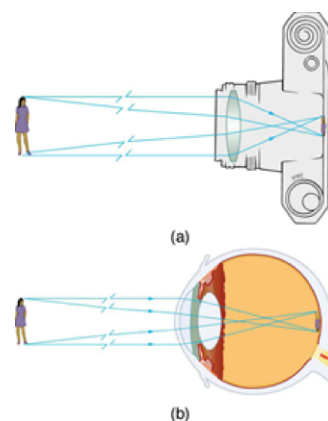
Figure 7.6: Rays demonstrate how an image is formed in (a) a camera, and (b) the human eye. Note that the eye contains a lens whose shape can be distorted by muscles to bring objects into focus.

- R_1 is taken to be positive if the front (upstream) surface is convex; R_2 is positive if the back (downstream) surface is concave. In the opposite cases, R is negative.

An alternate approach to summarizing the information about images and objects is to plot the thin lens equation in the form of $1/d_i$ vs $1/d_o$, as shown in Fig. 7.7. We start with the thin-lens equation,

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad \Longrightarrow \qquad \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

then realize what this says about a plot of $1/d_i$ vs $1/d_o$.

Now we can see that for any real object, a diverging lens will have a virtual object. For a converging lens, if $0 < d_o < f$, then $1/d_o > 1/f$, and the image will be virtual.

7.3 Aligning Optics

Finally, you should know the basics of caring for and aligning an optical system. In any optical system, such as your lenses or your camera,

- Make sure your optics are clean (please ask your instructor how to clean a lens so that you don't ruin the $50 lens), and use powder-free gloves when handling the lens;

- when setting up your imaging system be sure that each optical element is centered on the optical axis; you want light rays to hit your lens symmetrically both horizontally and
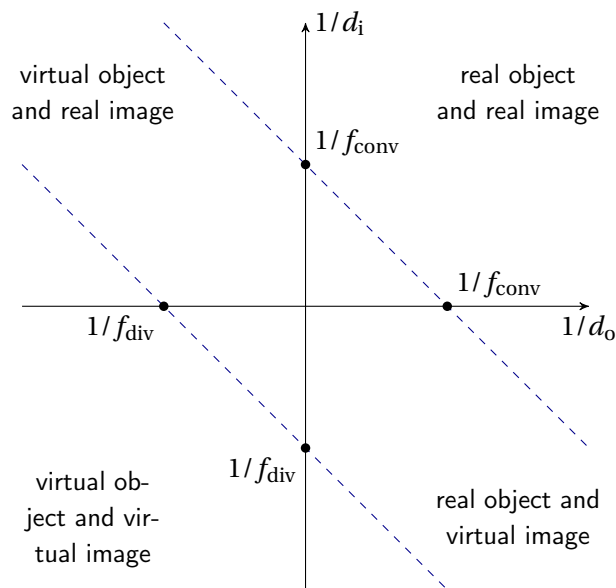
Figure 7.7: Plot of $1/d_i$ vs $1/d_o$.

vertically so that you don't introduce additional aberration to your image;

- it is important that the lenses are oriented so that they are perpendicular to the optical axis, otherwise you will introduce aberration in your image;

- the camera is one of your optical elements, so be sure that it is also centered along the beam axis (the sensor should be at the same height as the object and the lenses).

CAUTION: Any time you take the lens off of your camera, you are exposing your sensor to ambient air and dust and therefore the possibility of getting dirty. Dust sitting on your sensor will be noticeable in your exposures — much more so then dust on your lens — and is a big problem. Cleaning the sensor is problematic, so it's best to avoid getting dust or oil on the sensor in the first place. To this end, do not leave the lens off the camera any longer than is necessary, and try not to point the lens-less camera body up so as to avoid falling dust settling on the sensor. If you have any questions about camera care, please ask your instructor.

7.3.1   Image Capture Software

In this experiment you will need to use image capture software from Canon to transfer exposures from the camera to your computer over a USB interface. Canon supplies both a MacOS and a Windows version; install the version appropriate to your laptop.

MacOS

Warning: you need to follow these directions carefully. Do not double-click on the installer!

(1)   Go to the Support & Driver section of Canon's website (found at `https://www.usa.canon.com/internet/portal/us/home/support/details/cameras/support-dslr/eos-40d?tab=drivers#Z7_MQH8HIC0L88RB0AMD0F1Q42K25`).

(2)   Where it says **Enter Your Model Name**, put **EOS 40D** and hit Go.

(3)   Drivers & Software should already be selected. Now enter your operating system and select Software below. You should now have many choices. Download the latest updater for **EOS Utility**.

(4)   Unzip the download, open it, and drag the contents to your desktop.

(5)   Right click on this and select Show Package Contents.

(6)   Now, click on Contents, then Resources.

(7)   Delete the file that says info.datx.

(8)   Close the window and run the updater by double-clicking on the icon you dragged to your desktop (yes, now you can do it). You may get a message that says that the updater is damaged and cannot be run. If this happens, open system preferences, click on security and privacy and change it to allow apps downloaded from anywhere. Now run the updater and it should run like any other installer.

(9) If you are having trouble following this, there is a very helpful video at `http://vimeo.com/72683307`

Windows

Warning: you need to follow these directions carefully. Do not double-click on the installer!

(1) Go to the Support & Driver section of Canon's website (found at `https://www.usa.canon.com/internet/portal/us/home/support/details/cameras/support-dslr/eos-40d?tab=drivers#Z7_MQH8HIC0L88RB0AMD0F1Q42K25`).

(2) Where it says **Enter Your Model Name**, put **EOS 40D** and hit Go.

(3) Drivers & Software should already be selected. Now enter your operating system and select Software below. You should now have many choices. Download the latest updater for **EOS Utility**.

(4) Unzip the download, open it, and drag the contents to your desktop.

(5) Go to the start menu and type **regedit.exe**, then launch the editor.

(6) Click on **HKEY_LOCAL_MACHINE** and click on **SOFTWARE**.

(7) Right click on **Wow6432Node** and select **New Key**.

(8) At the bottom of the **Wow6432Node** folder there should be a folder that says **New Key #1**. Rename this **Canon**.

(9) Right click on this and select **New Key** and name it **EOS Utility**.

(10) You should now be able to run the download and it should install normally.

7.3.2 MATLAB Image Toolbox

You may need to use software to analyze properties of your exposure. For example, you may want to know how many pixels separate two bright spots in an exposure, or you might want to know the intensity count at a single pixel. To learn to do this please read through and become familiar with the "MATLAB Image Toolbox Tutorial". Or, you can use Igor Pro. Search the help system for information on image analysis.

7.3.3 Specifications for your Camera

The lens you are using with your camera has a focal length which can range from 28 mm to 135 mm, with minimum f-numbers of 3.5 and 5.6, respectively, depending on your zoom.

7.4   Notes on the apparatus

7.4.1   The Optical Rail and the Thin Lens Equation

In the exercise below, you may find it helpful to attach a light-tight snout to the camera body and/or cleverly place some black cardboard paper around the system to reduce the stray light that reaches your camera sensor. You might also put aluminum foil over the snout to further reduce stray light (create an approximately one-inch hole in the foil to let the desired light pass through to the sensor). Finally, you can adjust your lamp to increase or decrease the light incident on the objects you are trying to image.

7.4.2   Preliminary Exploration

Consider a single thin lens on your table and the thin lens equation. Image the business card at a variety of object distances from the lens and calculate the image positions that will result. (As an object, the dollar bill is even better than a business card; the dollar has lots of detail, interesting images and a surprising amount of color. It makes a rich target.) Check your calculations using the thin lens and the camera body (no lens attached) on the optical rail. Use Liveview in the EOS Utility so that you can see the image in real time. You'll probably want to run the camera in Av mode so that the shutter speed is automatically adjusted to the ambient light level.

Before you begin making quantitative measurements, you might want to play with the intensity and type of light that you use to illuminate your target. Does an incandescent light provide a different quality image than an LED? Do different lights create images with different colors? If you have a dollar bill as your target, does backlighting change the character of the image compared to front-side illumination?

Rotate slightly the lens in the post holder so that it is no longer perpendicular to the optical axis. Does the image distort? By how much do you have to rotate the lens before you affect the image?

Calculate and check the magnification of your system as well; does your calculated value match well with the value you measure? To determine the magnification of your optical system you might recall that the pixel on your sensor is square with sides of $5.7\,\mu m$ length.

Finally, place the camera's sensor along the rail at a distance away from the lens equal to the lens's focal length. What do you see?

7.4.3   Quantitative Measurements

Week 1: you should download the software and figure out how to use it. Then you should learn how to clean and mount lenses and work on aligning the optics. You should make at least one measurement of the focal length of a converging lens and check the magnification. With the knowledge of the first week you should be able to plan an efficient method of data collection for

following weeks.

Week 2: plot a graph like Fig. 7.7 for a converging lens to determine its focal length. Notice that you need at least two quadrants sampled so that your value for the focal length is interpolated not extrapolated.

Week 3: repeat the measurements for a graph like Fig. 7.7 for a diverging lens. To test your acumen, optimize a system to make a telescope or a microscope. Then characterize the device.

7.4.4 Hints

1. The riders on the optical rail can give useful information about the image and object distances but you need to figure out how to go from the location of the rider on the rail to the distance between the optical elements. Use the calipers to measure the distance between the object and the lens to find the offsets for that part of the system. To find the relationship between the rider for the camera and its sensor, you might use the magnification of one arrangement of the system.
2. To work with virtual images and objects you will need more than one lens. With a bit of planning, you will not need to know the focal length of the "extra" lens.
3. The data in this experiment are very sensitive to small systematic errors in the image and object positions. Think about how to estimate the size of these and adjust the data within acceptable limits. It may be that using the magnification will give the most accurate way to find these offsets.
4. Ray diagrams are extremely useful in figuring out how to arrange the lenses to create real or virtual images. The concept of a virtual object will make much more sense if you draw the ray diagrams for the rays that created it.
5. Before the third week you will need to do a bit of research to decide what kind of telescope you wish to construct.

7.5 Warm up exercises to do if you wish

(1) For each of the following lens systems calculate the total magnification and use the thin lens equation to calculate the final image position. What is the physical significance of a negative magnification? A negative image position?

   a. A 50-mm lens with an object distance of 100 mm. [Answer: 100 mm]

   b. A 200-mm lens with an object distance of 120 mm. [Answer: −300 mm]

   c. A −100-mm lens with an object distance of 300 mm. [Answer: −75 mm]

   d. A 400-mm lens is placed 250 mm from a 75-mm lens and the object distance is 900 mm from the 400-mm lens. [Answer: ≈ 65 mm, as measured form the second lens]

    e.   A 150-mm lens is placed 500 mm from a -50-mm lens and the object distance is 400 mm from the 150-mm lens. [Answer: ≈ -42 mm, as measured from the second lens]

(2) Calculate the focal length of a lens for blue light and for red light, where the lens has a thickness of 5 mm, a front surface radius of curvature of 180 mm and a back surface curvature of -250 mm. You will need to look up the dispersive properties of the lens.

(3) Your camera's lens has a focal length ranging from 28 mm – 135 mm, the ratio of the maximum and minimum determines the "zoom" on your lens. Over this range of focal lengths, what is the range of magnifications if we take the object distance to be much greater than either the focal length or the image distance?
[Answer: $|M| = i/o$, and fro the thin-lens equation, $i = of/(o-f)$. Substitute this value of i into the expression for magnification to get $M = f/(o-f)$ which, for $f \ll o$, gives approximately $M = f/o$. We consider the object distance to be unchanging, therefore M changes by the ratio of the focal lengths: $135/28 = 4.8$]

(4) Draw the ray diagrams for a converging lens with a virtual object and a diverging lens with a real image.