Experiment 8

Fresnel Coefficients

References

- *Optics* by Eugene Hecht, Chapter 4
- *Introduction to Modern Optics* by Grant Fowles, Chapter 2
- *Principles of Optics* by Max Born and Emil Wolf, Chapter 1
- *Optical Physics, 4^{th} edition* by Lipson, Lipson and Lipson, Chapter 5

8.1 Introduction

The ancients understood the law of reflection—that the angle of incidence was equal to the angle of reflection—but had a law of refraction that only worked near normal incidence. Willebrord Snel van Royen (1580–1626) was the first to figure out that $\sin\theta_1 \propto \sin\theta_2$, where the angles are measured with respect to the normal. Snel worked this out in 1621,[1] although he did not publish this result. René Descartes (1596–1650) gave the first published account in *La Dioptrique* (1637). In English-speaking countries, the relation

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{8.1}$$

is called **Snel's law**; in the French-speaking world, it is known as **Descartes' law**. You will explore a number of routes to deriving Snel's law, including Fermat's principle and Maxwell's equations.

But Snel's law is only part of the story: it doesn't determine how the incident beam should divide its energy between the reflected and refracted beams. Figuring out that mystery was long delayed by Newton's incomparable stature as *the* scientist of his age. His corpuscular theory of light did

[1] M. Born and E. Wolf, *Principles of Optics*, 7th edition (Cambridge, 1999) xxvi.

not lend itself easily to this purpose. The way forward came from Etienne Louis Malus (1775–1812), who discovered in 1808 that light reflected from a glass plate at a certain angle is polarized. Shortly thereafter in 1815, David Brewster (1781–1868) worked out that when the angle between the reflected and refracted beams is 90°, the reflected beam is polarized with its electric vector perpendicular to the plane of incidence. The angle of incidence that achieves this condition is called **Brewster's angle**. Jean-Augustin Fresnel's (1788–1827) wave theory was able not only to account for diffraction—leading to the outrageous, but confirmed, prediction of the Poisson bright spot—but also how the reflected intensity depends on polarization.

In this experiment, you will use optical rails, a semicircular prism, a linear polarizer, two wave-plates, and two photodiode detectors to investigate the laws of Snel, Malus, and Fresnel. The main goals are:

1. To determine the index of refraction of a material using Snel's law.
2. To investigate polarization of light and the effects of half- and quarter-wave plates.
3. To investigate the polarization dependence of reflection and transmission coefficients for a dielectric-air boundary.

The procedure that follows is vague by design, to encourage you to develop skill in aligning optics and in designing your own approaches to confirming these theoretical relationships.

8.2 Theoretical Background

8.2.1 Snel's Law

Snel's law describes the refraction of light at an interface between two dielectrics with indices of refraction n_1 and n_2. If the angle the incident beam makes with the normal to the local tangent

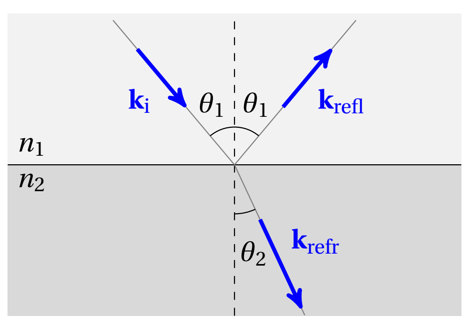


Figure 8.1: Geometry of refraction.

plane is θ_1 and the angle the transmitted beam makes with that normal is θ_2, as illustrated in Fig. 8.1, then the angles are related by Eq. (8.1).

Snel's law and the fact that the angle of incidence equals the angle of reflection are often introduced as facts of nature. To gain a deeper understanding of them, it is useful to examine four different approaches: the motion of wavefronts following Huygens' principle, a calculation using Fermat's principle of least time, the principles of quantum optics covered in Physics 51, and a derivation grounded in electromagnetic fields that applies boundary conditions at the interface between media that arise from Maxwell's equations. These approaches are all discussed in the reference in Hecht. The great advantage of the latter approach is that it incorporates the effect of polarization in a natural way.

You should understand the derivation of Snel's law from these different approaches.

8.2.2 Law of Malus

The law of Malus describes how the intensity of a linearly polarized beam of light traverses a perfect linear polarizer. The polarizer only transmits one component of the light wave's electric field, but since the intensity of the beam is proportional to the square of the electric field, the transmitted intensity is given by

$$I_{\text{trans}} = I_0 \cos^2 \phi \tag{8.2}$$

where ϕ is the angle between the pass axis of the polarizer and the electric vector of the incident beam.

8.2.3 Fresnel Coefficients

The Fresnel coefficients describe the amplitudes of reflected and transmitted electromagnetic waves at the interface between two dielectrics. The geometry is shown in Fig. 8.2. Recall that for an electromagnetic plane wave with angular frequency $\omega = 2\pi f$,

$$\mathbf{E}(x, y, z, t) = \mathbf{E}_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$$
$$\mathbf{E}_0 \perp \mathbf{k}$$
$$\omega = \frac{c}{n}|\mathbf{k}| \qquad \Longrightarrow \qquad f\lambda = \frac{c}{n} \qquad \Longrightarrow \qquad f\lambda_0 = c$$

where λ_0 is the wavelength in a vacuum. Because electromagnetic waves are transverse, the electric and magnetic fields are perpendicular to the direction of propagation. Any arbitrary $\mathbf{E}_0$ can be resolved into two components with directions shown in Fig. 8.2. The key feature is recognizing that the direction of propagation and the normal to the surface between the materials determine a plane called the "plane of incidence". In the figure, one material is below the plane of the interface and one is above.

The reflection and transmission coefficients are determined by applying the electromagnetic boundary conditions at the interface. Since the boundary conditions for the electric field depend on the direction of the electric field, the reflection and transmission coefficients for $\mathbf{E}_{||,\text{inc}}$ and $\mathbf{E}_{\perp,\text{inc}}$ are different. **You should go through the derivation of the reflection and transmission coefficients for the two polarizations.** For the special case in which the materials are non-magnetic, so that $\mu_1 = \mu_2 = 1$, the results are

$$r_\perp = \left(\frac{E_{\perp,\text{refl}}}{E_{\perp,\text{inc}}}\right) = \frac{n_\text{i}\cos\theta_\text{i} - n_\text{t}\cos\theta_\text{t}}{n_\text{i}\cos\theta_\text{i} + n_\text{t}\cos\theta_\text{t}} = -\frac{\sin(\theta_\text{i}-\theta_\text{t})}{\sin(\theta_\text{i}+\theta_\text{t})} \tag{8.3}$$

$$t_\perp = \left(\frac{E_{\perp,\text{trans}}}{E_{\perp,\text{inc}}}\right) = \frac{2n_\text{i}\cos\theta_\text{i}}{n_\text{i}\cos\theta_\text{i} + n_\text{t}\cos\theta_\text{t}} = \frac{2\sin\theta_\text{t}\cos\theta_\text{i}}{\sin(\theta_\text{i}+\theta_\text{t})} \tag{8.4}$$

$$r_{||} = \left(\frac{E_{||,\text{refl}}}{E_{||,\text{inc}}}\right) = \frac{n_\text{t}\cos\theta_\text{i} - n_\text{i}\cos\theta_\text{t}}{n_\text{t}\cos\theta_\text{i} + n_\text{i}\cos\theta_\text{t}} = \frac{\tan(\theta_\text{i}-\theta_\text{t})}{\tan(\theta_\text{i}+\theta_\text{t})} \tag{8.5}$$

$$t_{||} = \left(\frac{E_{||,\text{trans}}}{E_{||,\text{inc}}}\right) = \frac{2n_\text{i}\cos\theta_\text{i}}{n_\text{i}\cos\theta_\text{t} + n_\text{t}\cos\theta_\text{i}} = \frac{2\sin\theta_\text{t}\cos\theta_\text{i}}{\sin(\theta_\text{i}+\theta_\text{t})\cos(\theta_\text{i}-\theta_\text{t})} \tag{8.6}$$

where we can use Snel's law to relate θ_i and θ_t.

Note that these equations describe the electric field *amplitudes*, which are not the same as the intensities of the reflected or transmitted beams.

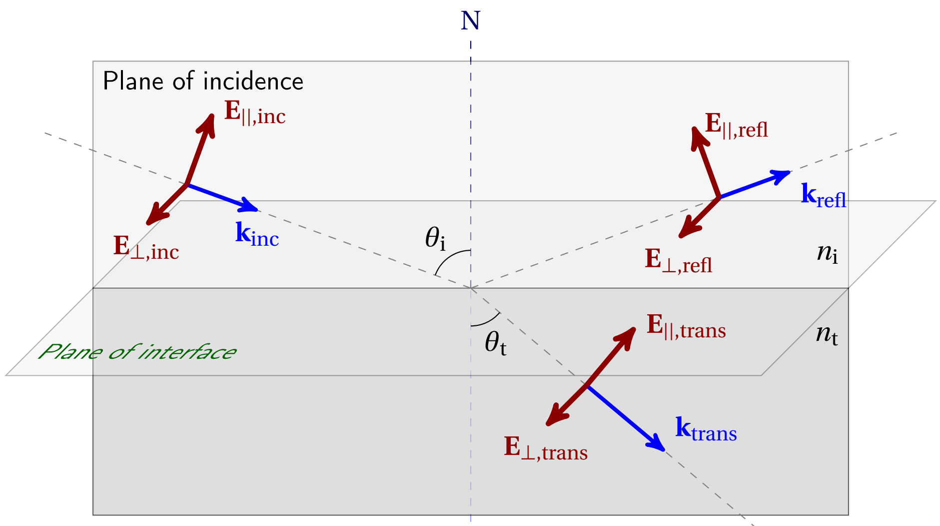


Figure 8.2: The geometry for the Fresnel coefficients. The incident electromagnetic wave can have polarization anywhere in the plane perpendicular to the direction of propagation. The incident field is thus a superposition of $\mathbf{E}_{||,\text{inc}}$ and $\mathbf{E}_{\perp,\text{inc}}$. In this figure, if the plane of incidence is the plane of the paper, then $\mathbf{E}_{\perp,\text{inc}}$ is out of the plane. Note: magnetic fields are not shown, but are orthogonal to both **k** and **E**. Boundary conditions derived from Maxwell's equations require that components of **E** and **B** that are parallel to the interface must be continuous, as must be the parallel component of **k**.

8.2.4 Brewster's Angle

Notice that the value for $r_{||}$ given in Eq. (8.5) goes to zero when $\theta_i + \theta_t = \pi/2$, which means that there is a particular angle of incidence where there will be no reflected light of the parallel polarization when that condition is met. This angle is called Brewster's angle.

8.2.5 Total Internal Reflection

If we examine Snel's law for the case that $n_1 > n_2$ then we see that there is a critical angle of incidence for which $\theta_t = \theta_2 = \pi/2$. At this condition, the transmission coefficients go to zero and all the light is reflected. What happens at larger angles of incidence?

8.2.6 Waveplates and Circular Polarization

In the discussion of Fresnel coefficients we resolved the incident electromagnetic wave into a superposition of linearly polarized components. Sometimes a more useful basis is that of left-hand and right-hand circularly polarized light. The action of a waveplate is to control the polarization of light and a waveplate can be used to change from linearly polarized to circularly polarized light or to change the orientation of linearly polarized light. The experimental setup includes a quarter and a half waveplate. Read about what these are and how they change the polarization of light.

8.3 Experimental Setup

The experimental setup is shown schematically in Fig. 8.3. It consists of a semicircular plastic prism on a rotation stage, a laser, two waveplates, a linear polarizer in a rotation mount and two photodiode detectors. These are mounted on a large board ruled with radial lines. The alignment procedure will be to ensure that the incident light hits the prism on the rotation axis. The prism is mounted on an *X*-*Y*-*R* stage so that it can be translated to make the alignment work. See Fig. 8.4.

Notice that the path of the light is most easily analyzed if the laser hits the flat face of the prism on the rotation axis and the center of the semicircle. Then the light refracted into the prism reaches the semicircular interface at normal incidence so does not refract on leaving the prism. The alignment procedure is designed to reach this position. One suggested process is detailed in Appendix A.

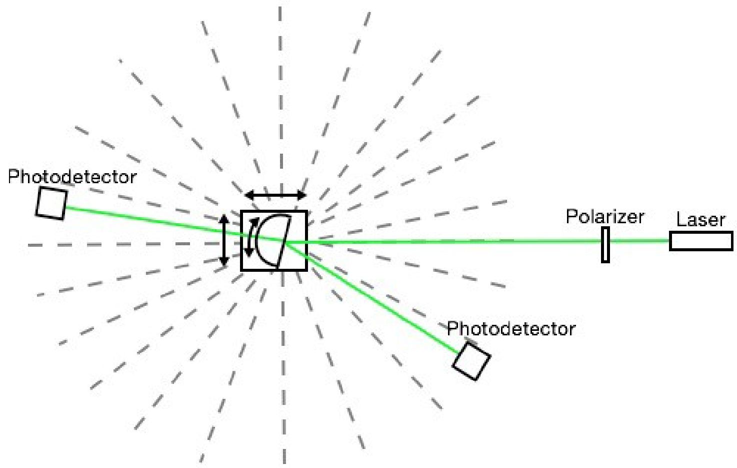


Figure 8.3: Schematic of the setup. The waveplates can be substituted for the polarizer or put in series with it. [Figure from Caleb Eades]

8.4 Qualitative Measurements

(to be answered with a combination of reading the specification sheets and using the equipment)

1. What type of detectors are supplied? How do they measure the intensity of the light? What do the scales on the top do to the signal?
2. What is the polarization state of the light from the laser?
3. Align the prism with the laser so the geometry for the rest of the experiment is convenient. Document the process you use and what you consider the figures of merit for a good alignment.

8.5 Quantitative Measurements

1. Determine if Snel's law applies to this system. If it does, then find the index of refraction of the prism. Observe total internal reflection and notice that a quick estimate of n can be found by making this one measurement.
2. Confirm that the detector and linear polarizer work as expected by taking measurements to confirm the Law of Malus.
3. Determine whether the quarter- and half-wave plates work as expected. At the end of these measurements you should be able to control the polarization of the laser beam.
4. Measure the reflected light from the prism as a function of incident angle for a couple of linear polarization directions. Analyze your data using the reflection coefficients to see if you

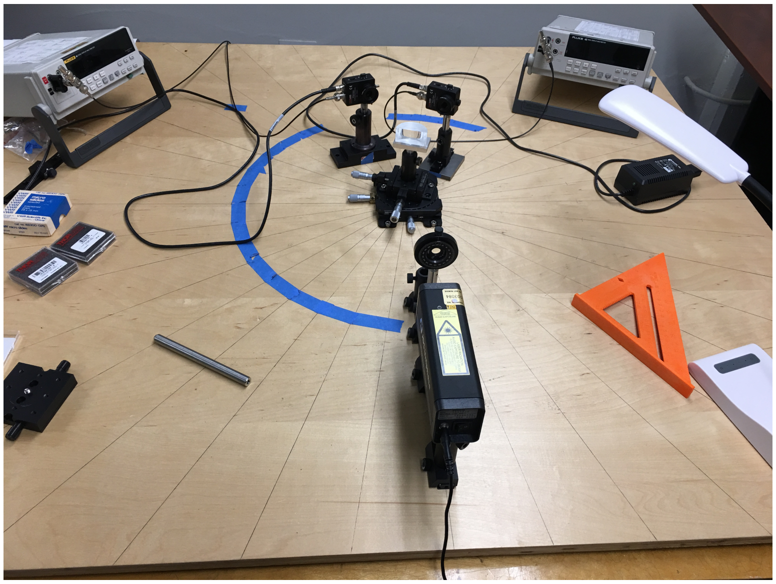


Figure 8.4: Photograph of the experimental setup.

have a consistent value for the index of refraction and if Equations (8.3)–(8.6) apply. Observe Brewster's angle.

5. Optional: Use another color of laser light to see if you can measure the dispersion in the prism. You can also investigate how well the waveplates work at this new wavelength.

6. Optional: Determine whether the transmitted intensities follow the predictions of Equations (8.3)–(8.6).

8.6 Appendix A: Alignment Suggestions

The following is slightly modified from the work by Caleb Eades.

We will now describe every step in the alignment procedure as it should be done sequentially when aligning the system.

1. The first step is ensuring that the laser beam (and hence the initial incident light) is going on a straight level line along one of the lines drawn on the table. The optimal way is to get a pair of irises on post mounts such that the irises are at the same height (note this can also be achieved using an index card in a card holder in a post mount with a dot on it with the dot centered over the post mount). Remove the prism temporarily and put one iris near the laser mounting strip and one far away, but make sure both are on the same line drawn on the table (the same one the laser will go on). Adjust the height of the laser mount and the positioning of the laser in the laser mount until the beam is going through the center

of both irises (or is hitting the dot on the index card in both the near and far field if using the index card). Use the set screw in the laser mount to adjust the positioning of the laser in the mount. Make sure that the laser is still at a height where it will hit the prism. If it is not, adjust the heights of the irises as appropriate and redo this step.

2. Put the polarizing optic in its mount on the same track as the laser diode and set it at a height such that the laser diode is roughly hitting the center of the optic (it should already be hitting the center horizontally based on step (i)). Twist the optic until the back reflection is into the laser diode as well as this can be arranged.

3. Place a stiff metal rectangular object of some width flat against the side of the square base of the prism-rotating stage. Barely unscrew the stage and then rotate holding the metal piece flat until the metal piece aligns exactly with the table line that is perpendicular to the table line the laser path is on (or the laser path line as it is a square). Screw the base back in. This ensures that the incident angle value read on the rotation stage has no offset (that is, the value you read is the actual incident angle value).

4. Put the prism in its mount and press it such that the flat face is against the flat part of the inset. Ensure that the rotation stage is set to 0° (that is, the flat face of the prism is perpendicular to the laser path). Now, use the micrometer perpendicular to the laser path to translate the rotation stage in the direction perpendicular to the path of the laser propagation until the beam is centered on the flat face of the prism. It is helpful here to use a neutral density filter to make the beam weaker and get a more precise centering. It is also helpful to look down the flat face of the prism and use the set screw hole to see where the laser should go (it should be centered on this hole as well).

5. (90°-90° test) One way to test whether the perpendicular position has been set correctly is to set the rotation stage to 90° and then look to make sure the beam is split in half, so if you hold an index card against the flat face of the prism, you should see half of the beam on the index card and half going onto the semicircular face of the prism. This should also be true when the rotation stage is set to 270°. If this is not the case, then try step 4 again.

6. (45°-45° test) Before this step, close the irises in front of the photodiode detectors all the way such that they have just a little hole. (Note you should adjust the height of the detectors such that the reflected beam is centered height-wise on the hole before continuing). Place the detectors on either side of the prism stage on the drawn line perpendicular to the laser path. Set the rotation stage (which should be at zero) to 45°. If the reflected light is not hitting one of the detectors exactly (to where the reflected beam is centered on the little hole), use the micrometer that is along the laser path to adjust the "z"-position of the stage until the beam is centered on the hole. Now set the rotation stage to 315°. If everything has been done properly, then the beam should be centered on the hole of the other detector. If the beam is not centered on the hole of the other detector, then something is off in the alignment. The potential issues are that the laser is not following the table line that it is mounted on, the square base of the rotation stage is not rotationally oriented properly, or

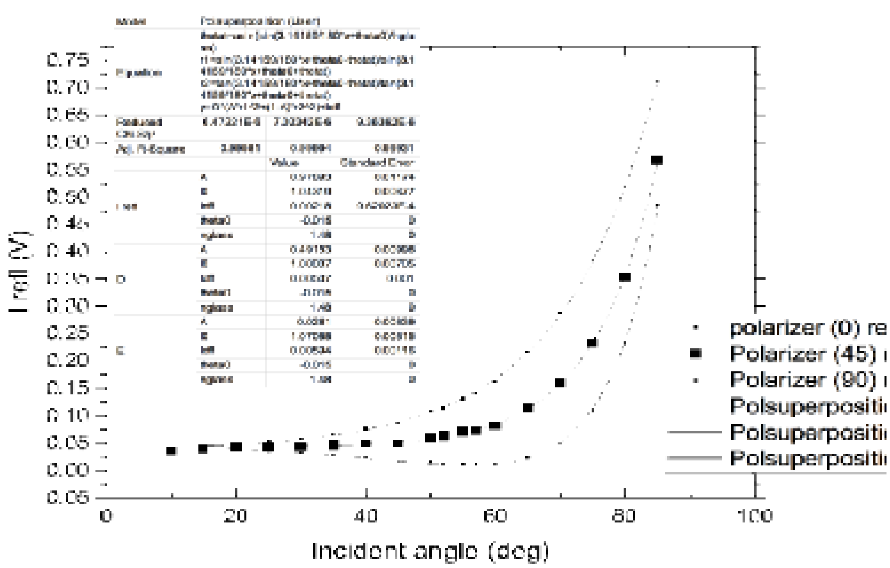
the beam is not hitting the center of the flat prism face. Repeat all the steps as necessary until the culprit is found.

If all these steps are done properly, then the data collected should be reasonably reliable (at least at the appropriate angles and with the center of the beam hitting the correct location on the center of the front flat face of the prism).

8.7 Appendix B: Sample Data for Fresnel Coefficients

Sample data for reflected light: A is the degree of polarization between 0 and 1. I fixed n and Origin happily found the polarization.

- Measure I_{refl} vs incident angle
- Fit with theta0 = −0.015 and nglass = 1.48
- Note that A = fraction of polarization
- thetat=asin(sin(3.14159/180*x+theta0)/nglass)
- r1=sin(3.14159/180*x+theta0-thetat)/sin(3.14159/180*x+theta0+thetat)
- r2=tan(3.14159/180*x+theta0-thetat)/tan(3.14159/180*x+theta0+thetat)
- y=I0*(A*r1^2+(1-A)*r2^2)+Ioff



If we examine the fitting coefficients we see good agreement for the values.

		Value	Standard Error
I refl	A	0.97893	0.01174
I refl	I0	1.04319	0.00677
I refl	Ioff	0.00218	9.62023E-4
I refl	theta0	-0.015	0
I refl	nglass	1.48	0
D	A	0.49153	0.00996
D	I0	1.00007	0.00705
D	Ioff	0.00507	0.001
D	theta0	-0.015	0
D	nglass	1.48	0
E	A	0.0261	0.00839
E	I0	1.07088	0.00815
E	Ioff	0.00594	0.00116
E	theta0	-0.015	0
E	nglass	1.48	0